Theory

of

Machines-II

(Module-1)

5th Semester B.Tech

Mechanical Engineering

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PCME4306	Theory of Machine-II	3-0-0	3 Credits

Course Objective:

1. To impart knowledge about dynamic analysis of mechanisms and balancing
2. To understand the principles of gyroscope and governors
3. To determine the static and dynamic forces for mechanical systems
4. To understand the principles of vibrations
5. To study tooth profiles of gears.

MODULE I (8 HOURS)

Motor Vehicle Steering Gears: - Davis Steering Gear& Ackermann Steering Gear.

Cams Design: Fundamental law of Cam, Cam Terminology, Classification of Cams and followers, Analysis of follower motions (Displacement, velocity, Acceleration and jerk) – Simple Harmonic, Uniform Velocity and Constant Acceleration & Retardation Types, Generation of Cam Profiles by Graphical Method.

MODULE – II (10 HOURS)

Turning Moment Diagram and Flywheel: Turning moment diagram. Turning moment diagrams for different types of engines, Fluctuation of energy and fluctuation of speed. Dynamic Theory of flywheel, Flywheel of an internal combustion engine and for a punch machine. Determination of flywheel size from Turning Moment Diagram.

Gears : Theory of shape and action of tooth properties and methods of generation of standard tooth profiles, Standard proportions, Force analysis, Path of contact, Arc of contact, Contact ratio, Interference and Undercutting, Methods for eliminating Interference, Minimum number of teeth to avoid interference.

MODULE III (10 HOURS)

Governors - Watt, Porter, Proell, Hartnell, Wilson Hartnell Governor. Performance parameters: Sensitiveness, Stability, Hunting, Isochronism. Governor Effort and Power, Controlling Force & Controlling Force Curve, Friction & insensitiveness, Comparison between governor and flywheel. Gyroscope: Introduction to Gyroscopes. Gyroscopic forces and Couple. Effect of Gyroscopic Couple on Aeroplanes,Gyroscopic stabilization of ship, Stability of Two Wheelers and Four Wheelers.

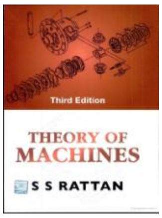
MODULE IV (12 HOURS)

Balancing: Static and Dynamic Balancing, Balancing of Single Rotating Mass by Balancing Masses in Same plane and in Different planes. Balancing of Several Rotating Masses rotating in same plane and in Different planes. Effect of Inertia Force due to Reciprocating Mass on Engine Frame, Partial balance of single cylinder engines. Primary and Secondary Balance of Multi-cylinder In-line Engines. Balancing of locomotive: variation of tractive force, swaying couple, and hammer blow. Direct and Reverse Crank method of balancing for radial engines. Balancing of V-engine. Balancing machines: Pivoted-Cradle Balancing Machine.

Vibrations: Introduction to Mechanical Vibration – Definitions, elements of vibratory system, Longitudinal, Torsional & Transverse Systems. Differential equations and solutions of motion for a coupled spring mass system. Determination of natural frequency of vibratory systems using energy method, equilibrium method and Rayleigh's method, Free and Forced Vibration of Un-damped and Damped Single Degree Freedom Systems, Logarithmic decrement, Magnification factor, Vibration isolation and transmissibility, whirling of shafts and Evaluation of Critical Speeds of shafts.

Text Books

1. Theory of Machines by S.S.Rattan, Tata MacGraw Hill
2. Theory of Machines by Thomas Bevan, Publisher: Pearson Education India
3. Theory of Machines and Mechanisms (India Edition) by John J. Uicker Jr., Gordon R. Pennock and Joseph E. Shigley, Oxford University Press
4. Mechanism and Machine Theory by J.S.Rao and R.V.Dukipatti, New Age International.
5. Theory of Machine by R.K Bansal,Laxmi publication.



Reference Books:

1 .Kinematics and Dynamics of Machinery by R.L.Norton,Tata MacGraw Hill
2. Kinematics & Dynamics of Machinery-Charles E. Wilson & J.Peter Saddler,Pearson Ed.
3.Theory of Machine by RS Khurmi, Publisher: S Chand
4.Mechanism and Machines by Amitav Ghosh & Mallik,Publisher: Affiliated East-West Press

Course outcomes:

On successful completion of the course the student will be able to,

1. Perform static and dynamic analysis of mechanisms
2. Understand the issues related to balancing of reciprocating and rotating machinery
3. Know the working of gyroscopes and flywheels
4. Have understanding about the effect of vibration.

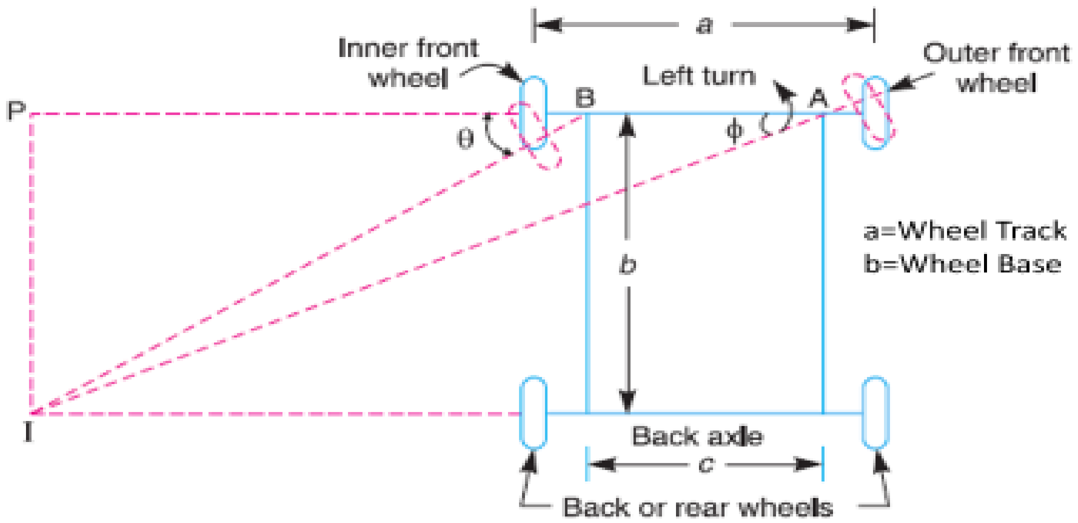
Chapter-1

Automobile Steering Gear

Steering system is the system which provides directional change in the performance of an automobile.

- This system converts rotary movement of the steering wheel into angular movement of the front wheels.
- It multiplies driver's effort by mechanical advantage, enabling him to turn the wheels easily.
- The steering gear mechanism is used to change the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.

Automobile Steering Gear



In order to avoid skidding the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels.

Now from the triangle IBP,

$$cot\theta = \frac{BP}{IP}$$

And from the triangle IAP,

$$cot\varphi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{IP}{BP} = \frac{C}{b} + cot\theta$$

$$\therefore cot\varphi - cot\theta = \frac{C}{b}$$

This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of wheels, when the vehicle takes a turn.

Question: *The ratio between the width of the front axle and that of the wheel base of a steering mechanism is 0.44. At the instant when the front inner wheel is turned by 18°, what should be the angle turned by the outer front wheel for perfect steering?*

Solution:

$$\frac{\mathrm{C}}{\mathrm{b}} = 0.44 \text{ and } \theta = 18°$$

As $$cot\varphi - cot\theta = \frac{C}{b}$$

$$cot\varphi - cot18° = 0.44$$

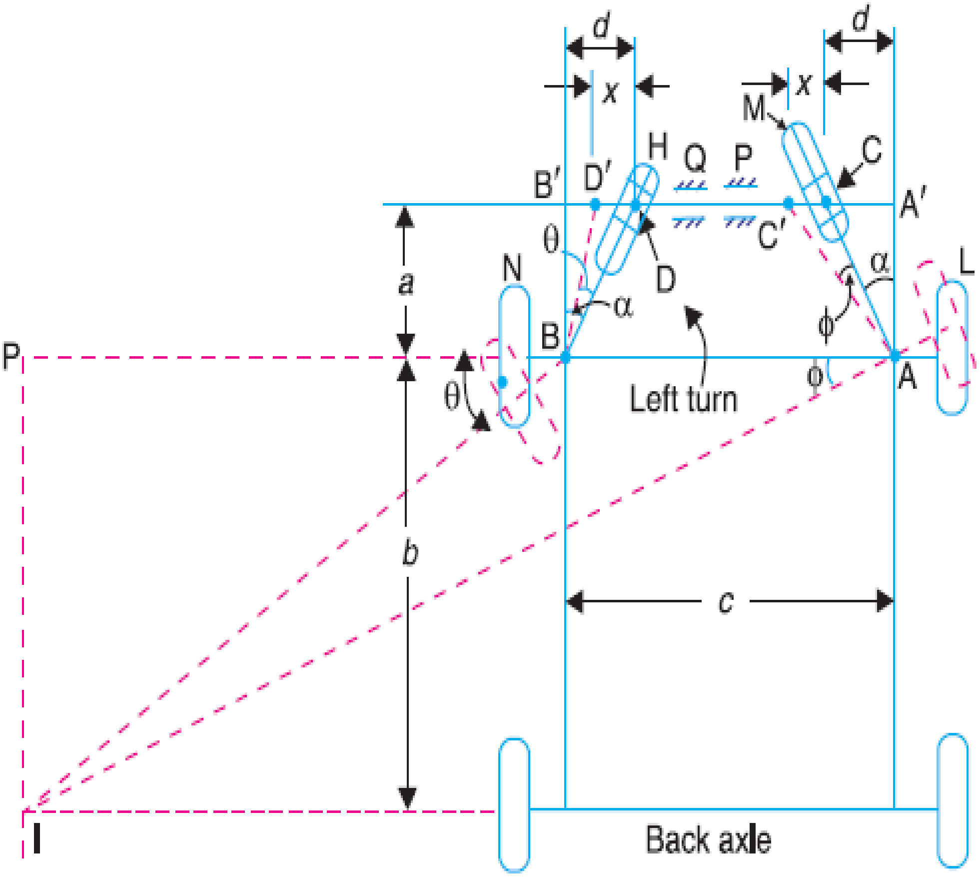
$$cot\varphi = 0.44 + 3.078 = 3.518$$

$$\varphi = 15.9°$$

Davis Steering Gear

The Davis gear mechanism consists of a cross link sliding *parallel to another link is connected to the stub* axles of the two front wheels by means of two similar bell crank levers pivoted.

- The cross link slides in the bearing and carries pins at its end. The slide blocks are pivoted on these pins and move with the turning of bell crank levers as the steering wheel is when the vehicle is running straight, the gear is said to be in its mid-position.
- The short arms are inclined an angle 90+α to their stub axles. The correct steering depends upon a suitable selection of cross-arm angle α.
- Davis steering gear is an exact steering gear mechanism. It has two sliding pairs and two turning pairs.
- In this mechanism, the slotted links are attached to the front wheel axle, which turn about two pivotal points. It has the rod and it is constrained to move in the direction of its length by the sliding two members. These constraints are connected to the slotted link by a sliding and a turning pair at each end.
- The main drawback in Davis steering mechanism is tearing and wear problem of sliding pairs. The drawbacks in Davis steering mechanism are overcome by Ackermann steering gear mechanism.



From the above figure

N and L are the front wheels

AB is the front Axle

HC is the track Arm

BN and AL is the stub Axles

BH and AC is the sliding pair

In triangle AA'C' $\tan(\alpha+\phi) = (d+x)/a$

In triangle AA'C $\tan\alpha = d/a$

In triangle BB'D' $\tan(\alpha-\theta) = (d-x)/a$

We know that,

$$\tan(\alpha+\phi) = \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}$$

$$\text{or} \quad \frac{d+x}{a} = \frac{\frac{d}{a}+tan\phi}{1-\frac{d}{a}\times tan\phi} = \frac{d+atan\phi}{a-dtan\phi}$$

$$(d + x)(a - dtan\phi) = a(d + atan\phi)$$

$$a.d - d^2tan\phi + a.x - d.xtan\phi = a.d + a^2tan\phi$$

$$tan\ \phi(a^2 + d^2 + d.x) = ax$$

$$\text{or} \quad tan\phi = \frac{ax}{a^2+d^2+d.x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Similarly, from $tan\ (\alpha - \theta) = \frac{d-x}{a}$, we get

$$tan\theta = \frac{ax}{a^2+d^2-d.x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

We know that for correct steering,

$$cot\phi - cot\theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{tan\phi} - \frac{1}{tan\theta} = \frac{c}{b}$$

$$\frac{a^2+d^2+d.x}{a.x} - \frac{a^2+d^2-d.x}{a.x} = \frac{c}{b} \quad \text{(from equation i \& ii)}$$

$$\frac{2d\ .x}{a.x} = \frac{c}{b} \quad \text{or} \quad \frac{2d}{a} = \frac{c}{b}$$

$$2tan\alpha = \frac{c}{b} \quad \text{or} \quad tan\alpha = \frac{c}{2b}$$

Question: *The track arm of a Davis steering gear is at a distance of 192 mm from the front main axle whereas the difference between their lengths is 96 mm. If the distance between steering pivots of the main axle is 1.4 m, determine the length of the chassis between the front and the rear wheels. Also, find the inclination of the track arms to the longitudinal axis of the wheels.*

Solution: $2d = 96$

d = 48

tan α = d/a = 48/192 = 0.25

Therefore, α = 14 °

Also, tanα = $\frac{c}{2b}$

Therefore, 0.25 = 1.4/2b

=> b = 2.8 m

Question: *The distance between the steering pivots of a Davis steering gear is 1.3m. The wheel base is 2.75m. What will be the inclination of the track arms to the longitudinal axis of the vehicle moving in a straight path?*

Solution:

c= 1.3m and b= 2.75m

tanα = $\frac{c}{2b} = 1.3/2 \times 2.75 = 0.236$

α= 13.3 or 13°18’

ACKERMAN STEERING GEAR MECHANISM

Fundamental equation of correct gearing.

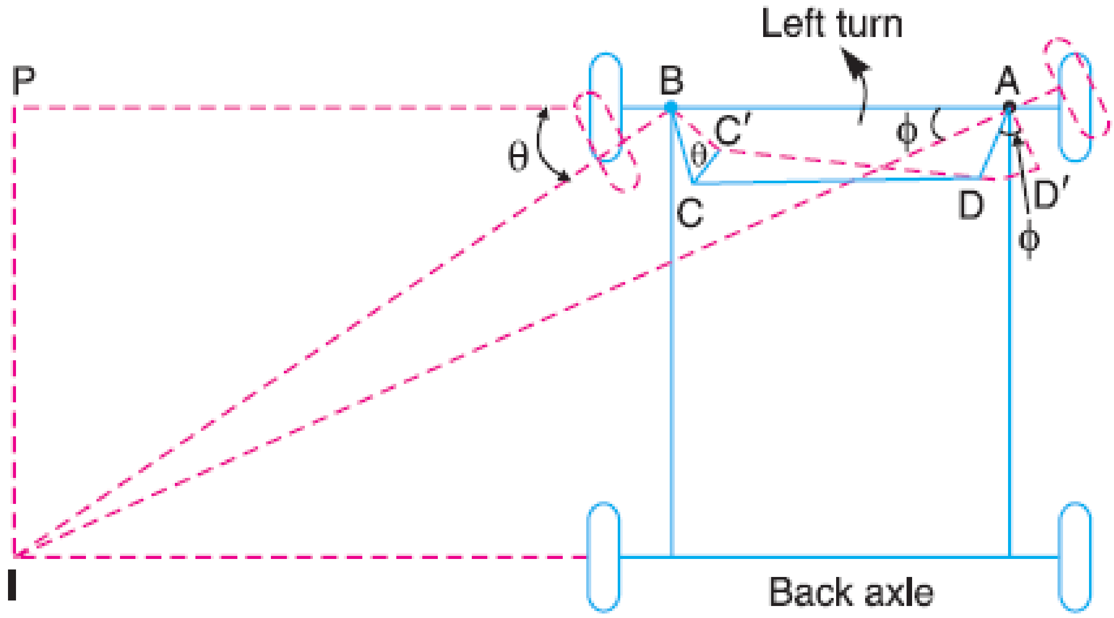
$$cot\varphi - cot\theta = \frac{c}{b}$$

Ackerman fulfils fundamental equation at the middle and two extreme positions.

In order to turn smoothly without any skidding/scrubbing of the tires, the axes of all wheels on vehicle must intersect at a single point; this point is present at the middle point of the back wheel axle.

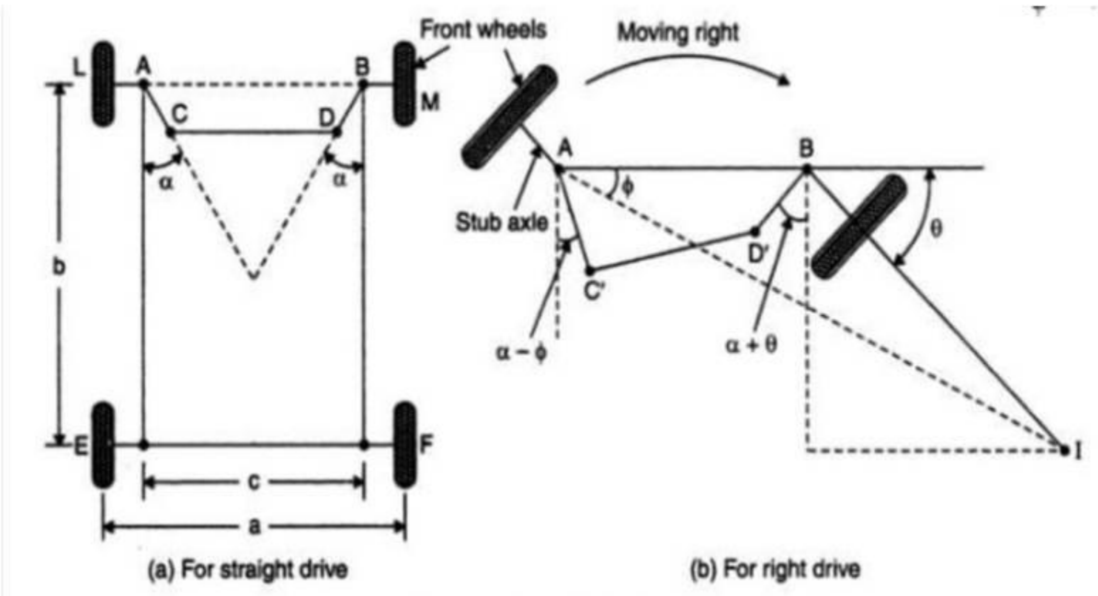
Ackerman mechanism is universally used because of its simplicity.

It consists of turning pairs. Whole mechanism of Ackerman steering gear is on back of front wheels.



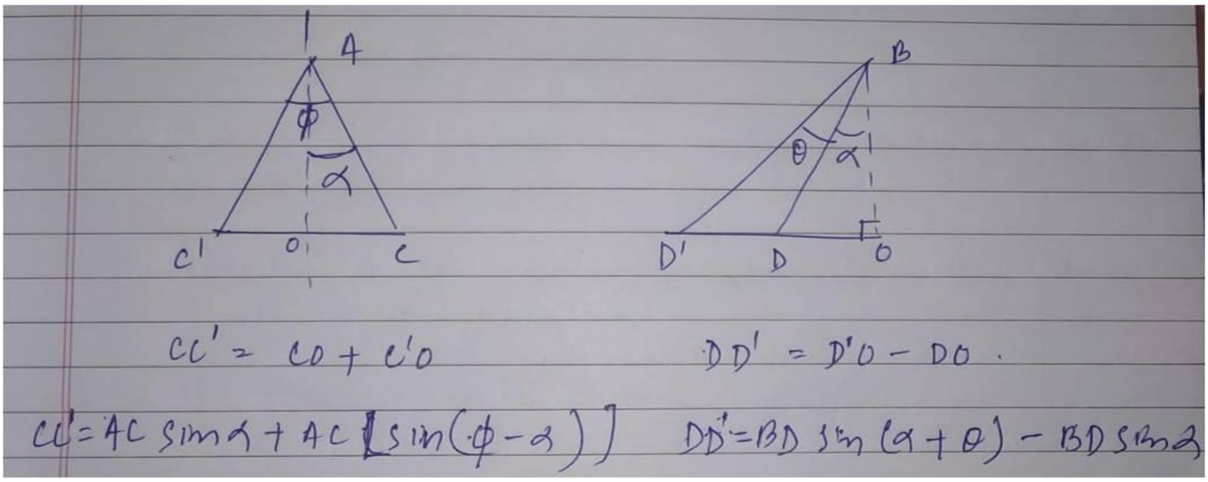
When the vehicle moves straight, the longer links AB and CD are parallel and shorter links BC and AD are inclined equally to the longitudinal axis of the vehicle.

When the vehicle moves left the position of the gear is shown by dotted lines.



Ackermann steering gear mechanism.

In this position the line of the front wheel axle intersect on back wheel axle at instantaneous centre (I) for correct steering. It consists of four link mechanism ABCD having four turning pairs. Two equal arms AC and BD are fixed to the stub axles AL and BM.



α is the angle of inclination of link AC and BD.

Projection of CC' = projection of DD' an front axle

BD [sin (α+ Θ)-Sin α] = AC[Sin α+Sin(φ - α)]

Since AC = BD

- $\sin(\alpha + \Theta) - \sin\alpha = \sin\alpha + \sin(\varphi - \alpha)$
- $\sin\alpha \cos\Theta + \cos\alpha \sin\Theta - \sin\alpha = \sin\alpha + \sin\varphi \cos\alpha - \cos\varphi \sin\alpha$
- $\sin\alpha(\cos\Theta + \cos\varphi - 2) = \cos\alpha(\sin\varphi - \sin\Theta)$
- $\sin\alpha/\cos = (\sin\varphi - \sin\Theta)/(\cos\Theta + \cos\varphi - 2)$
- $\tan\alpha = (\sin\varphi - \sin\Theta)/(\cos\Theta + \cos\varphi - 2)$

  Where,

  Θ and φ are the values of angles for correct steering.

  In Ackerman steering gear mechanism difference in slip angle minimise wheel skidding on turning.

  Due to inclination steering will automatically return to the initial state after turning.

Chapter-2

Cams

A cam is a rotating element that gives oscillating or reciprocating motion to the follower which is another element of this machine by direct contact. Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams.

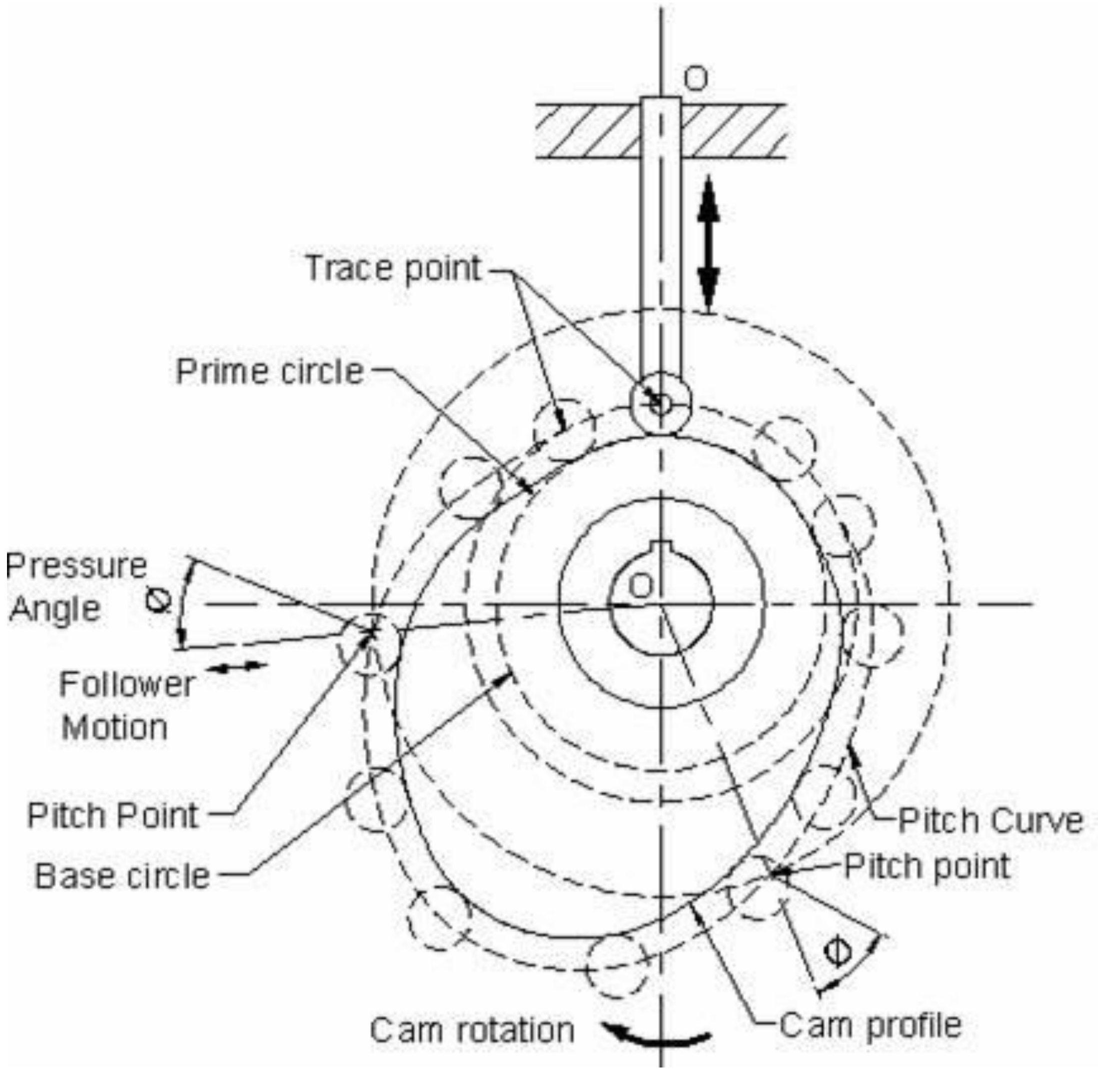
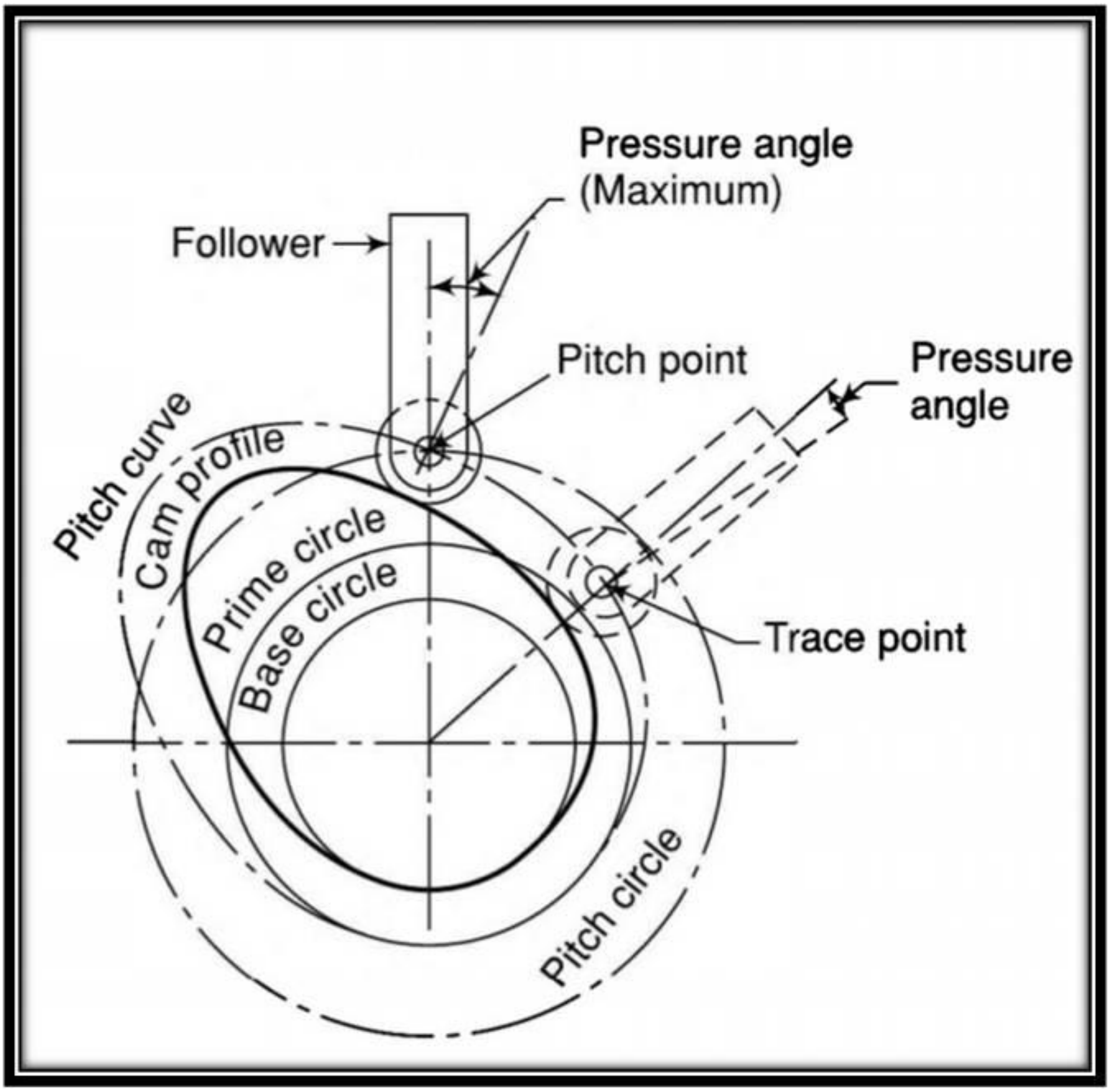


Fig.1

CAM TERMINOLOGY



(The above figure shows a radial cam with reciprocating roller follower).

The following terminology is used in reference to planar cam mechanism.

1. Cam profile

It is the actual working surface contour of cam. It is the surface of contact with the knife edge, roller surface or flat-faced follower.

2. Base circle

The smallest circle drawn tangent to a cam profile from the radial cam centre is known as *base circle*. The size of the cam depends upon the diameter of the base circle. The magnitude of pressure angle can be decreased by increasing the size of base circle.

3. Trace point

A theoretical point on the follower that traces the cam profile is known as *trace point.* In case of a knife-edged follower the edge of the knife, the centre of the roller for a roller follower and centre of the sphere of a spherical faced follower are the trace points.

4. Pitch curve

The path generated by the trace point as the follower moves relative to a stationary cam, is known as *pitch curve.* For a knife edge follower the pitch curve and the cam profile are same, but for a roller follower they are separated by the radius of the follower.

5. Pressure angle

The angle between the instantaneous direction of follower motion and a normal to the pitch curve is known as *pressure angle.* This angle represents the steepness of the cam profile. The pressure angle is in between 0 to ±30° for translating followers to avoid excessive slide load and if the follower is oscillating on a pivoted arm, a pressure angle up to about ±35° is acceptable. If the pressure angle is very high, it might jam the follower in the bearing. The magnitude of the pressure angle can be decreased by

1. Increasing the size of the base circle,

2. Decreasing the magnitude of follower displacement

3. Increasing the angle of cam rotation prescribed for the follower rise or fall.

6. Pitch point

A point on the pitch curve where the pressure angle is maximum is known as *pitch point.*

7. Pitch circle

A circle drawn from the centre of the cam through the pitch points is known as *pitch circle*. Pitch circle is concentric with the base circle.

8. Prime circle

The smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve is known as prime circle or reference circle .The prime and base circle are concentric. For a knife edged and a flat faced follower the prime circle is same as the base circle, but for roller follower the prime circle is larger than that of the base circle because of the roller radius.

9. Lift or Stroke

The maximum travel of the follower from the lowest position to the topmost position is known as *lift or stroke.*

10. Cam angle

The angle covered by the cam from its initial position is known as *cam angle.*

CLASSIFICATION OF CAM AND FOLLOWER

Types of CAM:

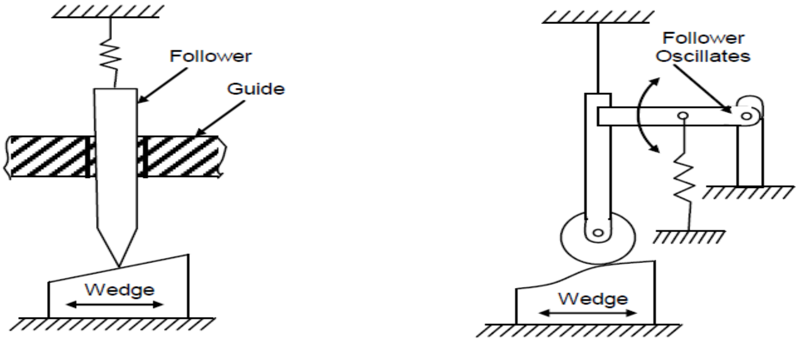
CAMS are classified according to following criteria:

1. Shape.
2. Follower movement.
3. According to manner of constraint of follower.

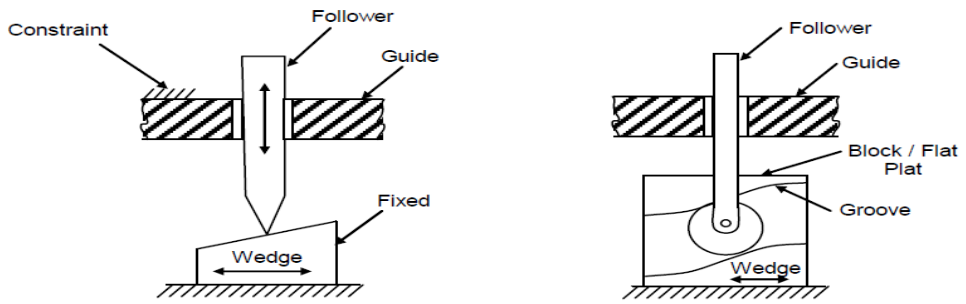
ACCORDING TO SHAPE:

1. WEDGE AND FLAT CAMS:

A wedge cam consists of a wedge "W" and a follower "F". Wedge "W" has a translational motion and "F" can either translate or oscillate. This type of cams usually consists of a spring which is used to maintain the contact between the cam and the follower. The cam in figure (c) is stationary and the guide "G" or follower constraint causes the relative motion between them. In fig. (D) a flat plate with groove can be used in place of wedge and the follower is held on that groove. As in fig (D) spring is not used, hence it is a positive drive.

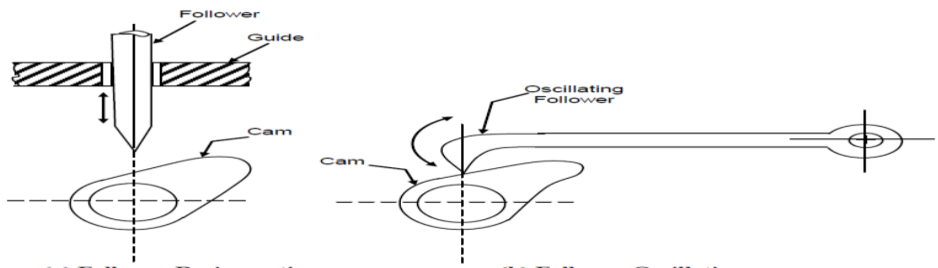


(a) Wedge Cam (b) Wedge Cam

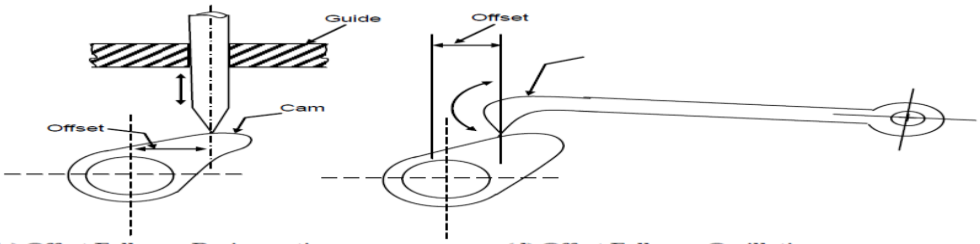


2. RADIAL CAMS OR DISC CAMS:

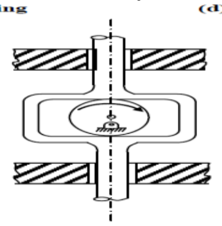
Radial cams are those types of cams in which follower move radially from the centre of rotation of cam. These are very popular due to its compactness and simplicity. A **radial cam** converts a rotational motion into a translational motion of the follower.



(a) Follower Reciprocating (b) Follower Oscillating



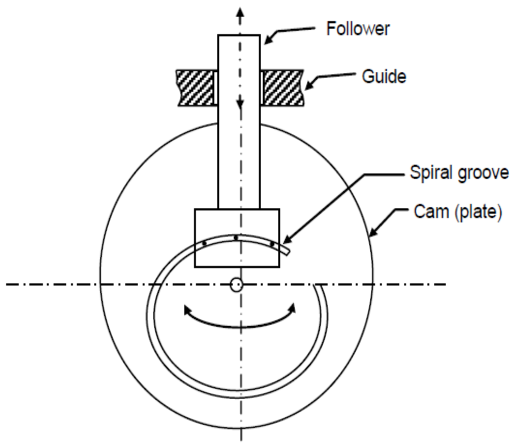
(c) Offset Follower Reciprocating (d) Offset Follower Oscillating



(e)

3. SPIRAL CAMS:

It is a face cam which consists of a groove cut in form of a spiral. This spiral groove consists of teeth which are meshed with a pin gear follower. Here, in this type of cams the velocity of the follower is proportional to the radial distance of the groove from the axis of the cam. The usage of these types to cams is limited as the cam has to reverse its direction to reset the position of the follower. It is used mostly in computers. The movement of the follower depend upon the radial distance of the grooves which is measure from the axis of the cam.



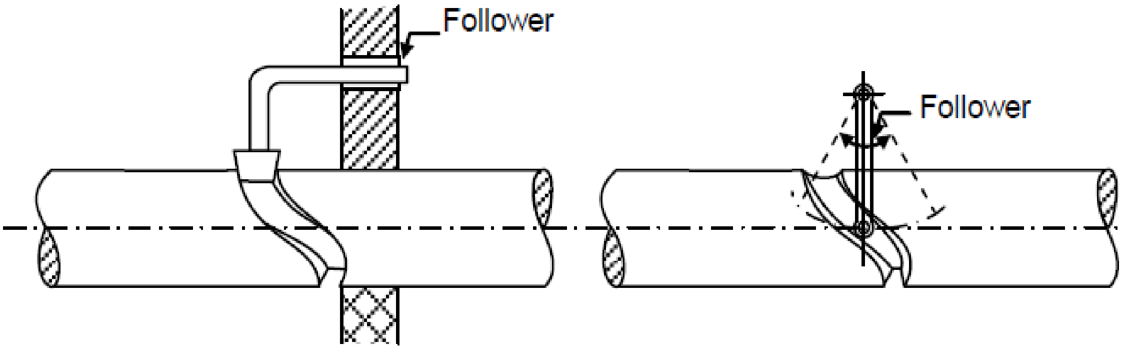
4. CYLINDRICAL CAMS:

These types of cam consist of a circumferential contour cut in the surface which rotates about its axis. Usually the follower motion can be of two types as follows:

- In the first type, there is groove cut on the surface of the cam and the roller follower has a constrained or positive oscillating motion.

- In the second type, an end of the cylinder which is the working surface is an end cam and the spring loaded follower has a translational motion along or parallel to the axis of the rotating cylinder.

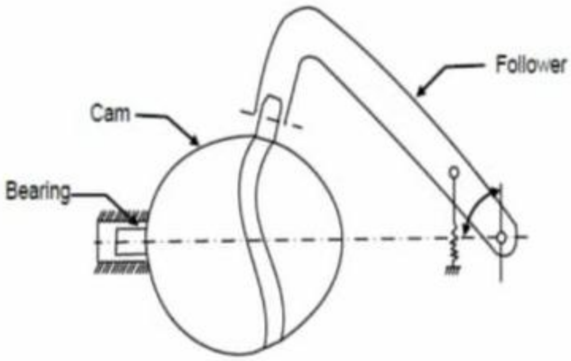
These types of cams are also known as barrel or drum cams.



(a) Follower Reciprocates **(b) Follower Oscillates**

5. SPHERICAL CAM:

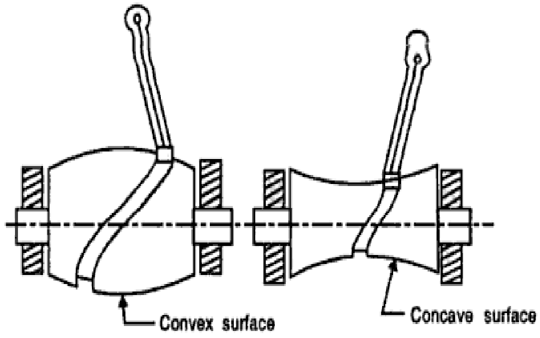
Spherical cam is in the form of **spherical** shape on which groove is made. Follower has surface contact with this groove. As **cam** rotate follower oscillate about a fixed point.



Spherical Cams

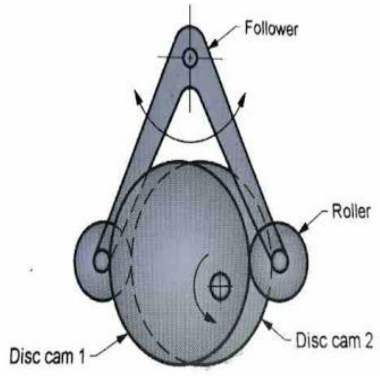
6. GLOBOIDAL CAM:

A globoidal cam may have either concave or convex surface and a circumferential contour is cut on the surface. The follower in these cams has an oscillatory motion.



7. CONJUGATE CAM:

In this type two discs are connected to each other and axes of discs are offset by the distance, to form cam profile. Follower has surface contact with cam profile by two rollers. As the cam rotate follower follow it.



Conjugate cam

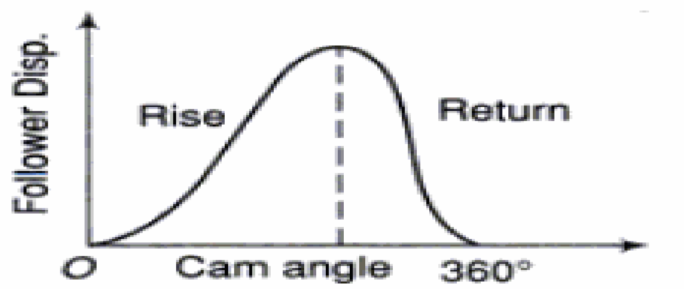
CLASSIFICATION ACCORDING TO MOVEMENT OF THE FOLLOWER:

The motions of the followers are distinguished from each other by the dwells, rises and return they have.

- ***Rise of a cam:*** The motion of the cam which tends to lift the follower is known as the rise motion.
- ***Dwell of a cam:*** The rotation of the cam for which the follower is stationary at its position is known as dwell of the cam.
- ***Return of a cam:*** The motion (rotation) of the cam for which the follower tends to move its original position is known as the return motion of the cam.

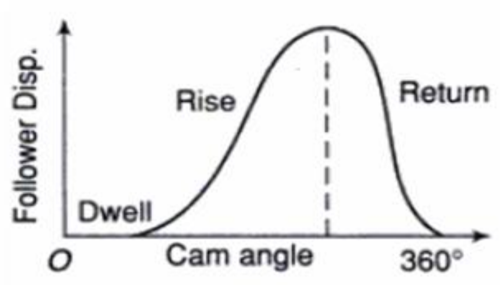
CAMS ARE CLASSIFIED ACCORDING TO THE MOTIONS OF THE FOLLOWERS;

1. *Rise-Return-Rise* (R-R-R): In this, there is alternate rise and return of the follower with no periods of dwell. Its use is very limited in the industry. The follower has a linear or an angular displacement.



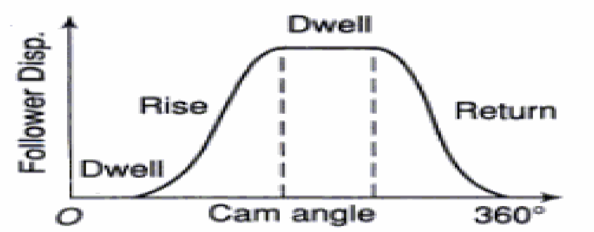
[**Figure: Rise-Return-Rise Follower motion**]

2. ***Dwell-Rise-Return-Dwell (D-R-R-D):*** In such a type of cam, there is rise and return of the follower after a dwell. This type is used more frequently than the R-R-R type of cam.



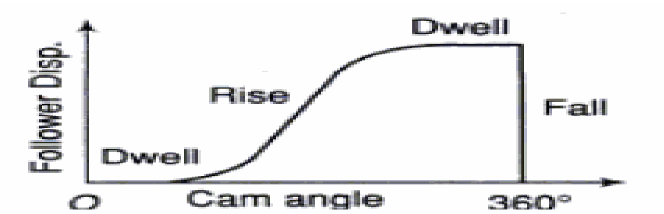
[**Figure: Dwell-Rise-Return-Dwell Follower motion]**

3. *Dwell-Rise-Dwell-Return-Dwell (D-R-D-R-D):* It is the most widely used type of cam. The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell.



[**Figure: Dwell-Rise-Dwell-Return-Dwell Follower motion]**

4. Dwell-Rise-Dwell (D-R-D): The dwelling of the cam is followed by dwell, rise and then dwell of the follower.



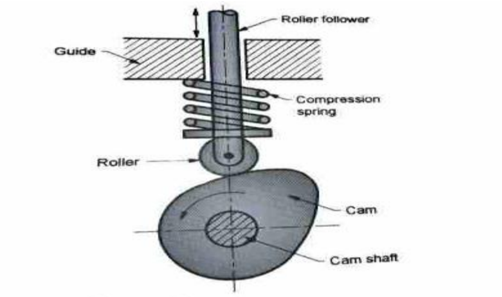
[**Figure: Dwell-Rise-Dwell (D-R-D) Follower Motion]**

ACCORDING TO MANNER OF CONSTRAINT OF THE FOLLOWER:

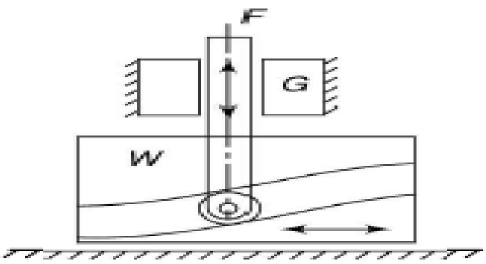
For proper function of cam and follower mechanism, it is important to keep surface contact between cam profile and follower at all time of rotation and any speed. For this need spring force, gravity force or positive drives are use in cam and follower mechanism. On this basis cams are classified by type of constraint of follower and these are as follow.

1. **PRELOADED SPRING CAM:**

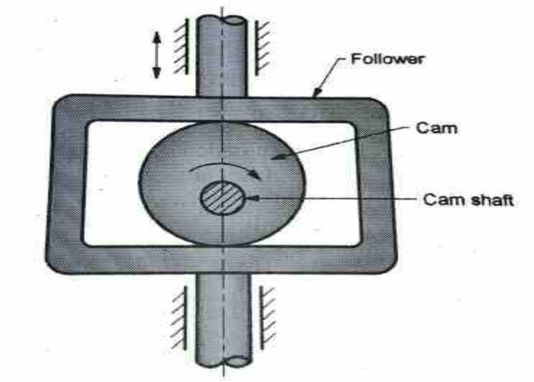
   In this type of cam spring force is used. Compression spring is preloaded into follower in such a way that it maintains surface contact all time.



Preloaded spring cam



2. **POSITIVE DRIVE CAM:**



Positive drive cam

In this type of cam, it does not require any external force. The follower is design in such a way that it does not require any external force to maintain contact between cam and follower.

3. GRAVITY CAM:

Contact between the cam and follower is maintained due to the gravity or own weight of the follower, it is known as gravity cam.

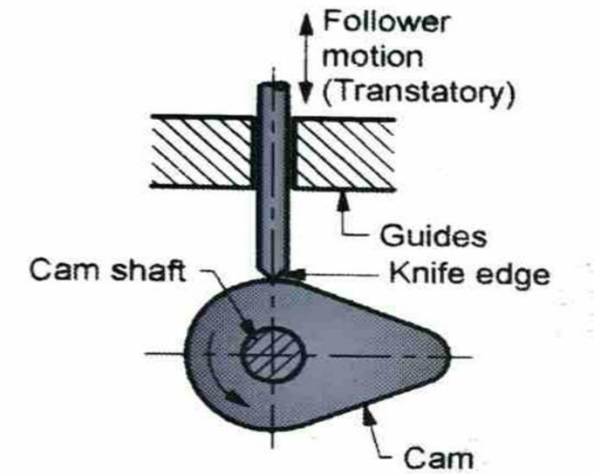
TYPES OF FOLLOWER:

- Follower is a machine element which is used in cam and follower mechanism.
- Cam and follower mechanism is used in various machines to transmit motion from cam to follower in a particular direction.
- Cam and follower have line contact thus constitute a higher pair.
- Types of follower are in below.

ACCORDING TO SURFACE CONTACT:-

1. KNIFE EDGE FOLLOWER:

When the contacting end of the follower is sharp knife edge, then it is called knife edge follower. This is the simplest among all of the followers and these kinds of

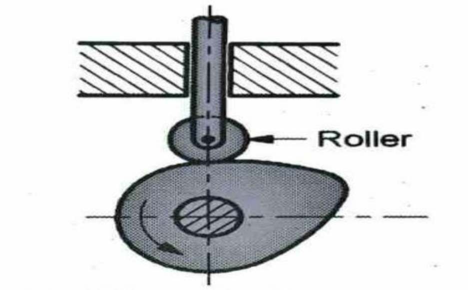


Knife edge follower

followers aren't in use in case of fast application, because of its sharp edge.

2. ROLLER FOLLOWER:

When the contacting end of the follower is a roller it is called roller follower. This type of follower is mainly used in high speed operation because it has a smooth

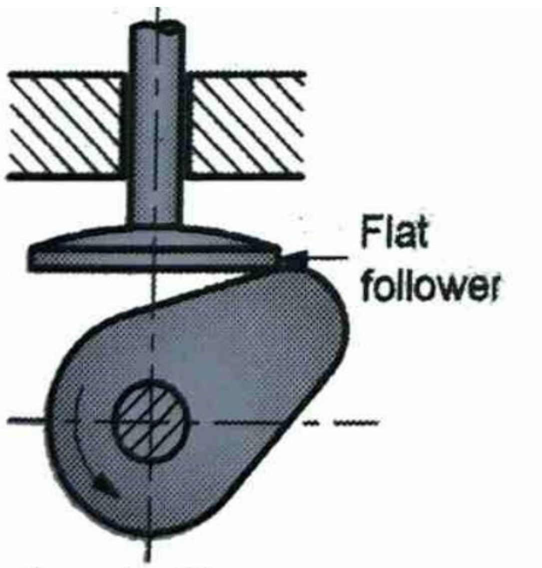


Roller follower

contact with the surface. This type of follower has less wear and tear as compared to other followers.

3. FLAT FACE OR MUSHROOM FOLLOWER:-

When the contacting end of the follower is perfectly flat face it is called flat face or mushroom follower. This

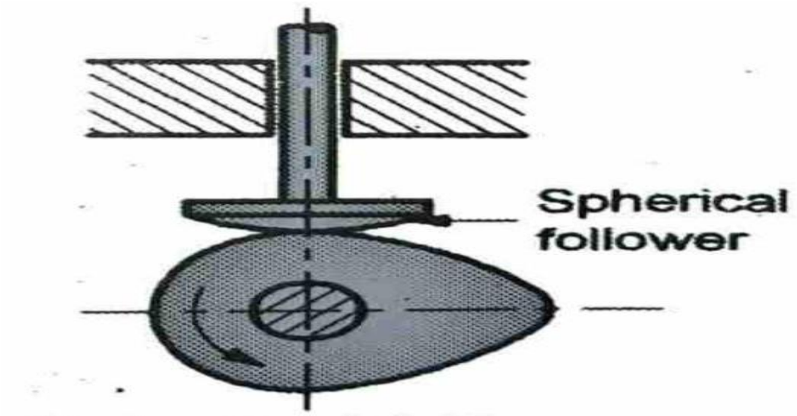


Flat follower

type of follower looks like a flat surface with an irregular cam. This type of cam is used when the space is limited and this follower can resist more side thrust.

4. SPHERICAL FOLLOWER:

When the contacting end of the follower is of spherical shape, it is called spherical follower. This type of follower has a curved but regular follower as well as cam. This is a modification of flat faced follower.



Spherical follower

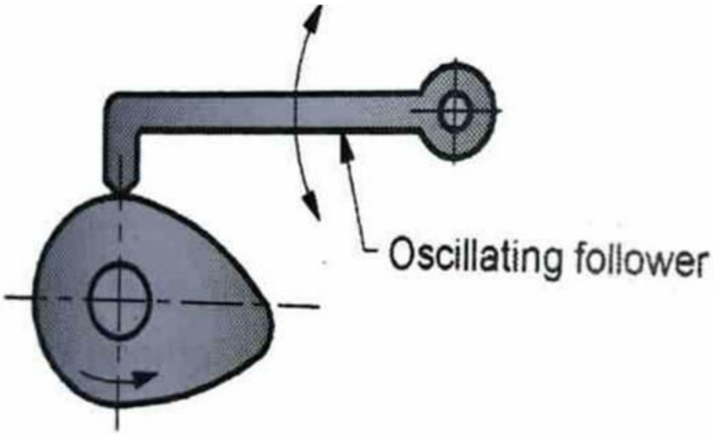
ACCORDING TO THE MOTION OF FOLLOWER:

1. Reciprocating or translating follower:

When the follower reciprocates in guides as the cam rotates uniformly, it is called reciprocating or translating follower.

2. Oscillating or rotating follower:-

When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower it is Oscillating or rotating follower.



oscillatory motion type follower

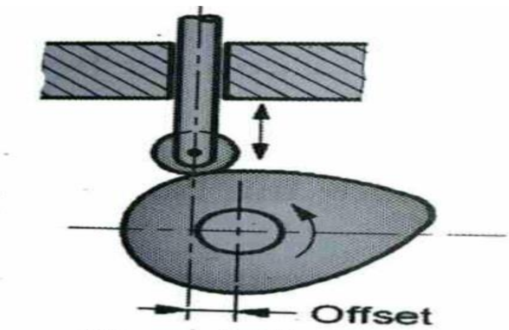
ACCORDING TO THE PATH OF MOTION OF THE FOLLOWER:

1. RADIAL FOLLOWER :

When the motion of the follower is along an axis passing through the centre of cam it is known as radial follower.

2. OFFSET FOLLOWER

When the motion of the follower is along an axis away from the axis of the cam centre it is known as offset follower.



offset follower

FOLLOWER DISPLACEMENT PROGRAMMING

Cam follower systems are designed to achieve a desired oscillatory motion. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam.

Follower displacement can be plotted against the angular displacement θ of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along the y-axis and angular displacement θ of the cam is plotted along x-axis. Displacement diagrams are basic requirements for the construction of cam profiles.

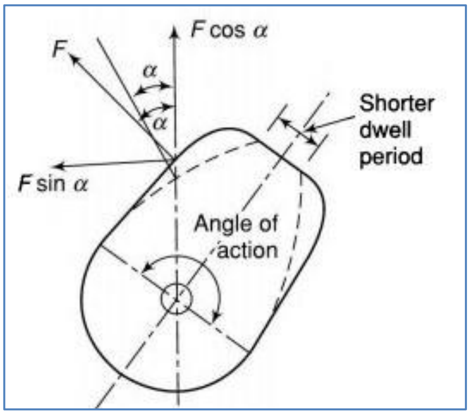
The following terms are used with reference to the angular motion of the cam:

- **Angel of ascent ($Ø_a$) :-** It is the angle through which the cam turns during the time the follower rises.
- **Angel of dwell (δ):-** The angle of dwell is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.
- **Angel of decent ($Ø_d$):-** It is the angle through which the cam turns during the time the follower returns to the initial position.

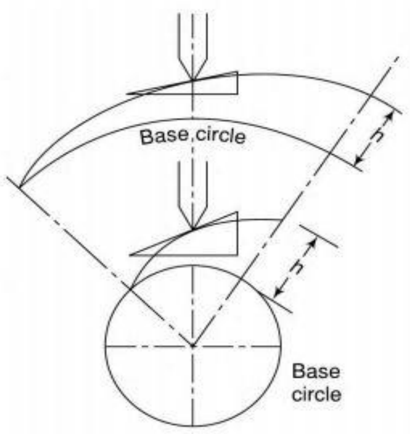
- **Angel of action**:- The angle of action is the total angle moved by the cam during the time, between the beginning of rise and the end of the return of the follower.

To satisfy the given requirements of the follower displacement, a programme can be made keeping in view the following points:

1. In case of high-speed mechanism produce vibrations of the system, this is undesirable for a follower motion. Though it is very difficult to completely eliminate jerk, efforts are to be made to keep it within tolerable limits.
2. The force exerted by a cam on the follower is always normal to the surface of the cam at the point of contact. The vertical component (F cos α) lifts the follower whereas the horizontal component (F sin α) exerts lateral pressure on the bearing.



   In order to reduce the lateral pressure or F sin α, α in reduced velocity of the follower and more time for the same rise. This also reduces the dwell period for a fixed angle of action.
3. The size of the base circle controls the pressure angle. The increase in the base circle diameter increase the



length of the arc of the circle upon which the wedge (the raised portion) is to be made. A short wedge for a given rise requires a steep rise or a higher pressure angle, thus increasing the lateral force.

Derivatives of Follower Motion

The derivatives of follower motion can be

- Kinematic (with respect to θ, relate to geometry of cam system)
- Physical (with respect to time, relate to motion of the follower)

Kinematic Derivatives

A displacement diagram of the follower is plotted with the cam angle θ as the abscissa and the follower linear or angular motion as the ordinate.

Mathematically, if s is the displacement of the follower, then

$$s = s(\theta)$$

Differentiating it with respect to θ provides the first derivative,

$$s'(\theta)= ds/d\theta$$

It represents the slope or the steepness of the displacement curve at each position of the cam angle.

The second derivative is represented by

$$s''(\theta) = d^2s/d\theta^2$$

This derivative is related to the related to the radius of the cam at different points along its profile and is in inverse position.

The next derivative can also be taken if desired

$$s'''(\theta) = d^3 s / d\theta^3$$

It is not easy to describe it geometrically.

Physical Derivatives

We have,

$$s = s(\theta) \text{ and } \theta = \theta(t)$$

Taking the first derivative with respect to time,

$$s' = ds/dt = ds/d\theta \times d\theta/dt = \omega \times ds/d\theta$$

which represents the velocity of the follower.

The second derivative is

$$s'' = d^2 s/ dt^2 = \omega^2 \times d^2 / d\theta^2.$$

It represents the acceleration of the follower. A higher value of acceleration means a higher inertia force.

A third derivative is known as the jerk.

$$s''' = d^3s/ dt^3 = \omega^3 \times d^3s/d\theta^3$$

For smooth movement of the follower, even the high values of the jerk are undesirable in case of high speed cams.

High Speed Cams :

:-Inertia force of the follower is obtained when mass of real follower is multiplied by acceleration.

:-This force is always felt at the contact point of the follower with cam surface.

:-An acceleration curve with abrupt changes exerts abrupt stresses on the cam surfaces accompanied by detrimental effect such as surface wear and noise.

:-All this may lead to an early failure of cam system.

:-So it is very important to consider velocity and acceleration curve while choosing displacement diagram.

:-Higher the speed of the cam, higher is the need for smooth curves.

:-At very high speed, jerk is to made continuous.

SIMPLE HARMONIC MOTION

DISPLACEMENT DIAGRAM-

In a cam follower system, the motion of the follower is very important. The follower displacement is plotted against the cam angle (ø) and it called displacement diagram. The displacement of the follower is plotted along the y-axis and the cam angle (ø) is plotted along the x-axis. From the displacement diagram, velocity and acceleration can also be plotted for different cam angle (ø). The displacement, velocity and acceleration are plotted for 1 rotation of the cam, i.e. 360^0. Displacement diagram are also used in construction of cam profile.

Here we will construct displacement diagram when the follower undergo S.H.M (Simple Harmonic Motion).

Steps to draw the Displacement Diagram of follower when it undergoes S.H.M –

Consider that x-axis represents the cam angle (ø) & y-axis represents the linear or angular motion of the follower.

Step-1

-Mark the value of lift (in cm.) on the y-axis. Find the middle point of the lift.

-Draw a half circle keeping the middle point of the lift as centre. Joining the two end points of the lift.

Step-2

Divide the angle made by the half circle into equal angles according to the value of lift & mark it as 0, 1, 2, 3 according up to the last point.

Step-3

On the y-axis take the scale, for simplicity take 10^0 as 1 unit. Motion of the follower can be divided into 4 parts. *i.e.*

1. Outstroke
2. Dwell – 1
3. Return stroke
4. Dwell – 2

1. Outstroke

- 1st mark the value of outstroke from the original point & mark it as 1, 2, 3, ….. up to the outstroke end point.
- Then join the line 1 on the x-axis & 1 on the y-axis, 2 on the x-axis & 2 on the y-axis and so on.
- Then we will get the point (1,1), (2,2), (3,3) & so on.
- Now join the points by hand.

2. Dwell – 1

- Similarly from the end point of the outstroke mark the value of dwell-1.
- As in dwell condition the follower does not move, so no curve will draw.

3. Return stroke

- Mark the value of return stroke from the end point of the Dwell – 1 & number the path from maximum value of return stroke to 0.
- Then join point 1 on x-axis & point 1 on the y-axis, point 2 on x-axis and point 2 on y-axis and so on.
- So we will get the points (1, 1), (2,2), (3,30, (4,4) and so on.

- Now join the parts by hand.

4. Dwell – 2

- When the value of Dwell 2 is not given subtract the sum of outstroke, Dwell 1, return stroke from 360°to get the value of Dwell 2.
- And if it is not possible to mark on the x-axis then draw a curved line at some distance from the end point of return stroke.

The whole diagram represents the displacement of the follower.

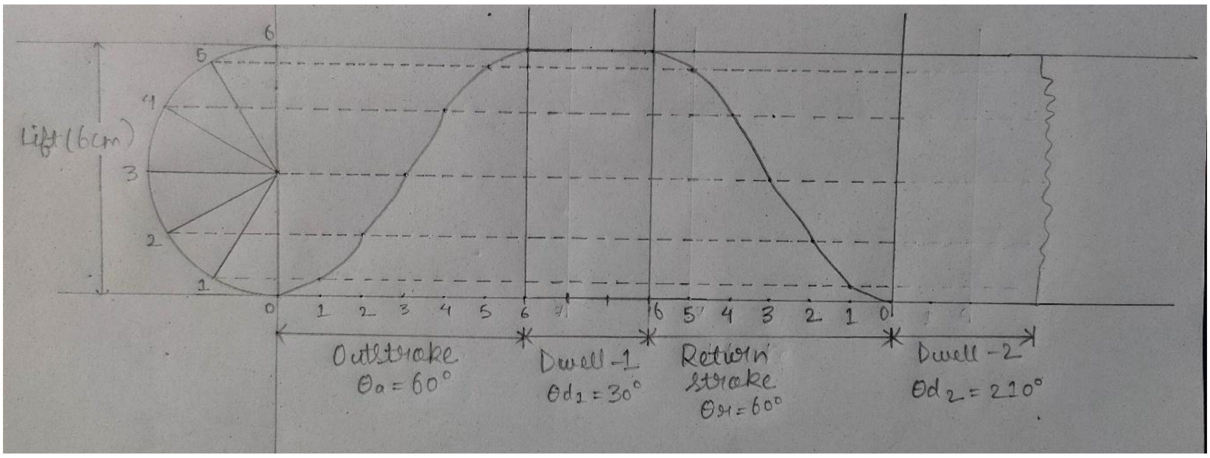
For example:

Let, outstroke – 60°

Lift – 6cm

Dwell 1 – 30°

Return stroke – 60°



So, dwell 2 = 360° – (60°+30°+60°) = 210°

Example:

lift = 50mm; out stroke with SHM for 60° cam rotation; dwell for 45° cam rotation; return stroke with SHM, for 90° cam rotation; dwell for the remaining period.

Solution:

i. Draw a semi circle with cam rise (or full) at the diameter. This is usually known as the harmonic (semi) circle. Divide this semi circle into 6 equal arcs.

ii. Divide the cam displacement interval into 6 equal divisions.

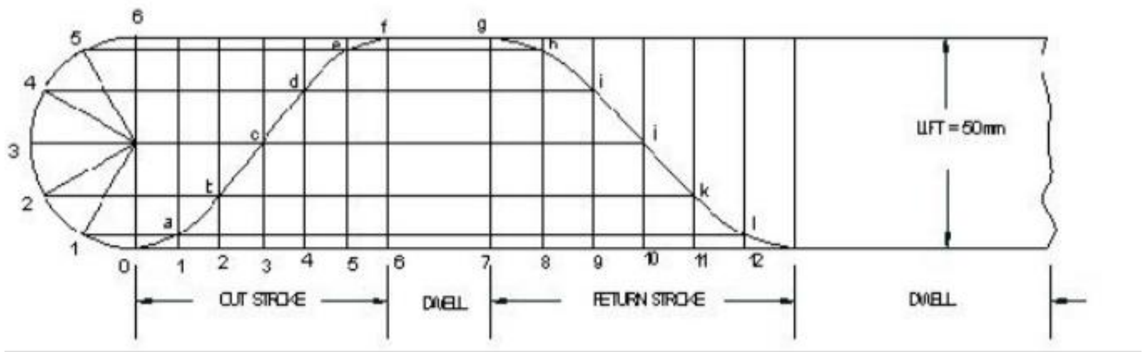
iii. Project the intercepts of the harmonic semicircle to the corresponding divisions of the cam displacement interval.

iv. Join the points with a smooth curve to obtain the required harmonic curve.

In figure,

Scale: 10°=1cm (for outstroke)

Scale: 10°=1.5cm (for return stroke)



MOTION OF THE FOLLOWER

The follower, during its travel, may have one of the following motions.

1. Uniform velocity
2. Simple harmonic motion
3. Uniform acceleration and retardation, and
4. Cycloidal motion

UNIFORM VELOCITY

In uniform velocity motion, the cam rotates and the follower moves with uniform velocity. As the follower moves with uniform velocity, the displacement of the follower is directly proportional to the displacement of the cam (i.e. $S \alpha \theta$). Hence the slope of the displacement curve is constant.

Let's take the follower displacement(s) on the ordinate (y-axis), cam angle (θ) on abscissa (y-axis).

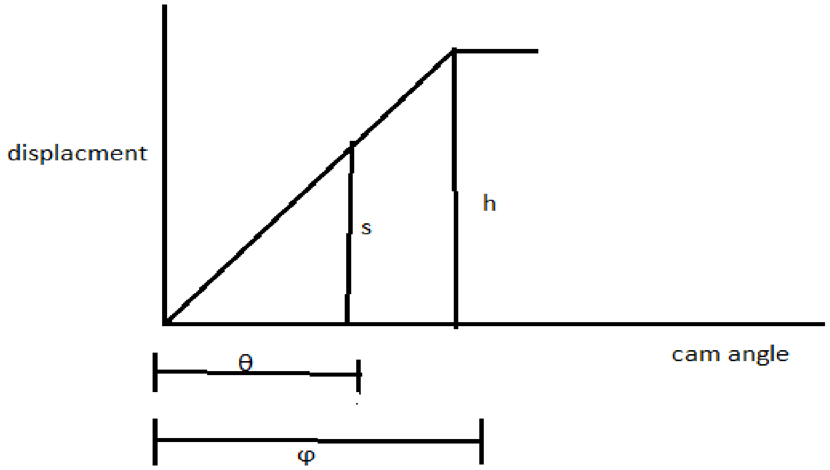
Let $S/\theta = h/\phi = \text{const.} = c = \text{slope}$

Where, c= constant of proportionality

S=instantaneous displacement

H= max. displacement of the follower

Φ=max. cam rotation angle



Again, $S/\theta = h/\phi$

⇨ $S = h\,\theta/\phi$

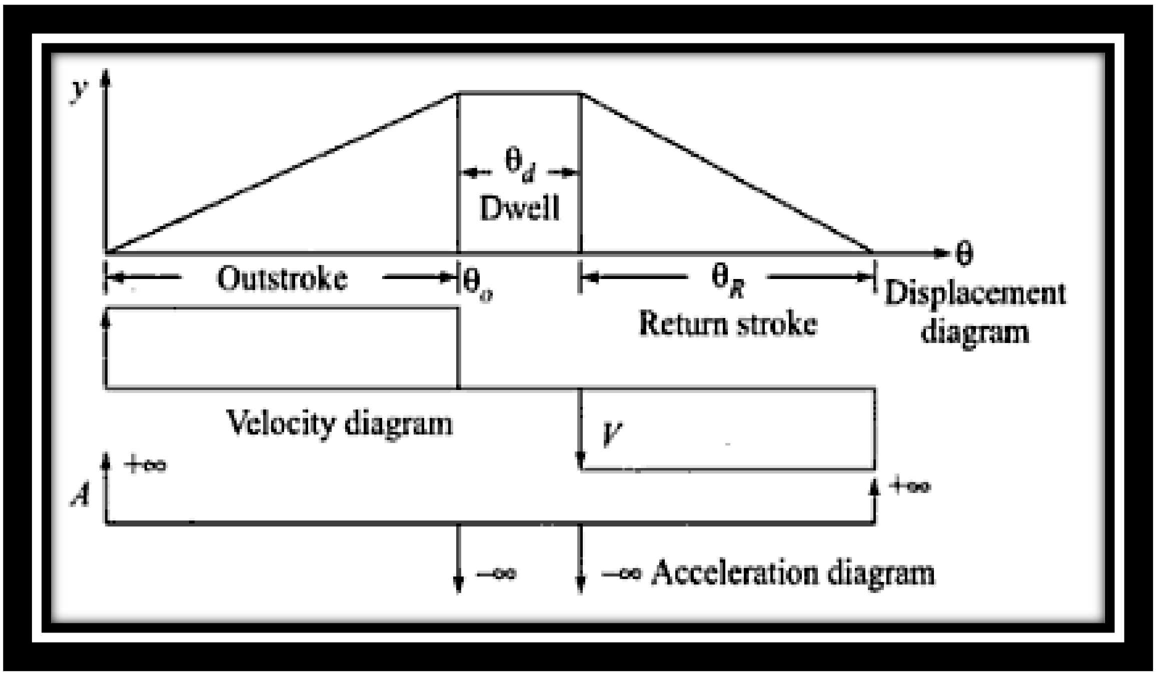
⇨ $S = h\,\omega/\phi$ (as $\theta = \omega\, t$)

Therefore, Velocity, $v = ds/dt = h\,\omega/\phi = \text{const.}$

Acceleration, $f = dv/dt = 0$

As the slope of the displacement curve is constant the velocity will remain constant during rise and fall of the displacement and in the dwell time as the slope is zero so the velocity will be zero.

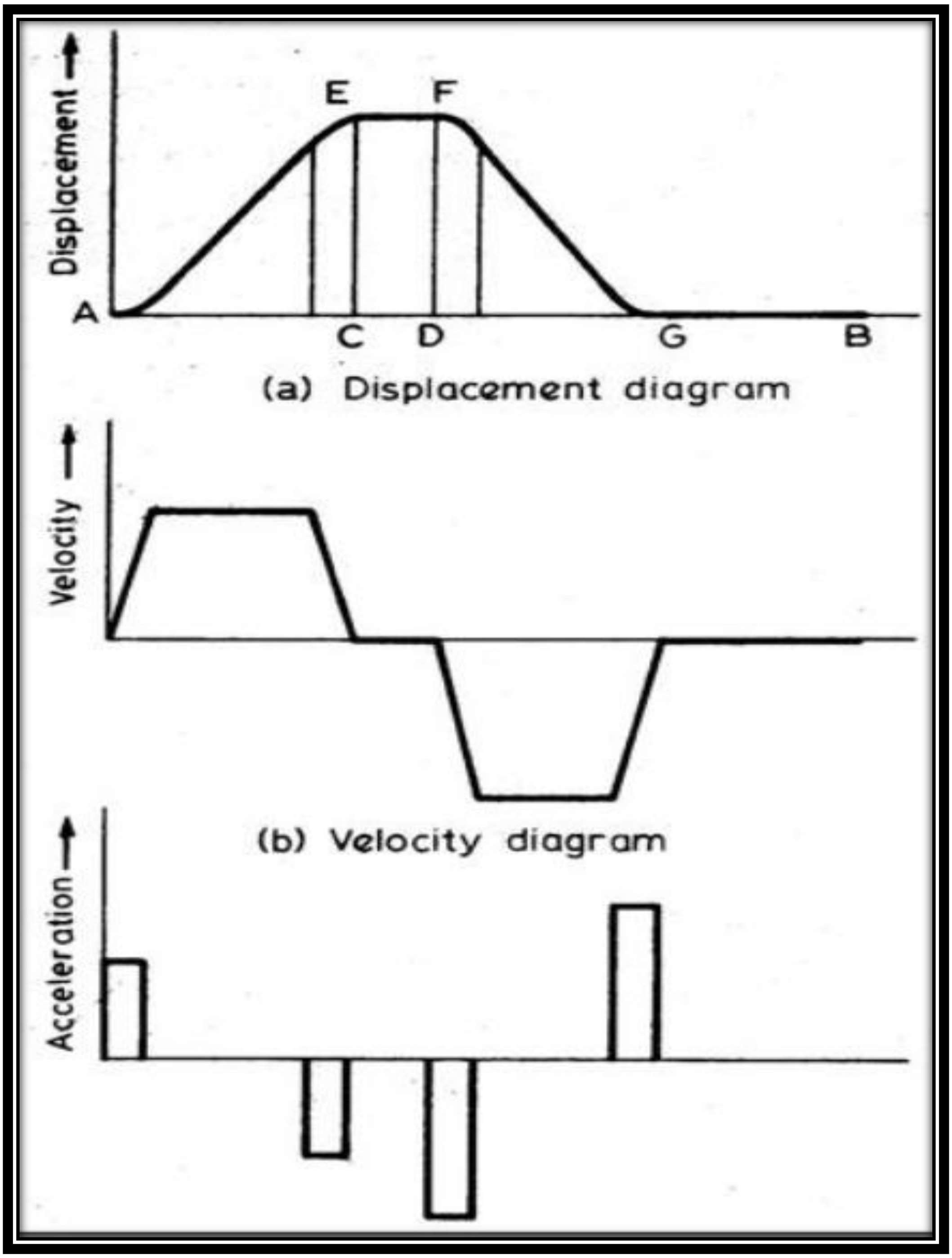
The acceleration during the rise and fall is zero. But the acceleration at the beginning and end of the rise and fall is infinite. This happens due to sudden rise or fall of velocity with very negligible time (almost zero). As the acceleration is infinite the inertia force will also become infinite within very short duration of time. But this is practically impossible.



Motion with uniform velocity

This problem can be eleminated by replacing the sharp corners of the displacment curve with some parabolic or round curve. We take parabolic curve because it gives very low acceleration.

Again, as the displacment curve is round in nature it gives gradual increase of velocity and hence gives a constant acceleration to the follower.



Modified motion with uniform velocity

EXAMPLE :

A cam is to be given the following motion to a knife edge follower:

1: outstroke during 60^0 of the cam rotation;

2: Dwell for next 30^0 of cam rotation ;

3: Return stroke during next 60^0 of cam rotation;

4: Dwell for remaininf 210^0 of cam rotation.

The stroke of the follower is 6cm. the folloewer moves with uniform velocity during outsroke and return stroke. Draw the displacment diagram fo the follower.

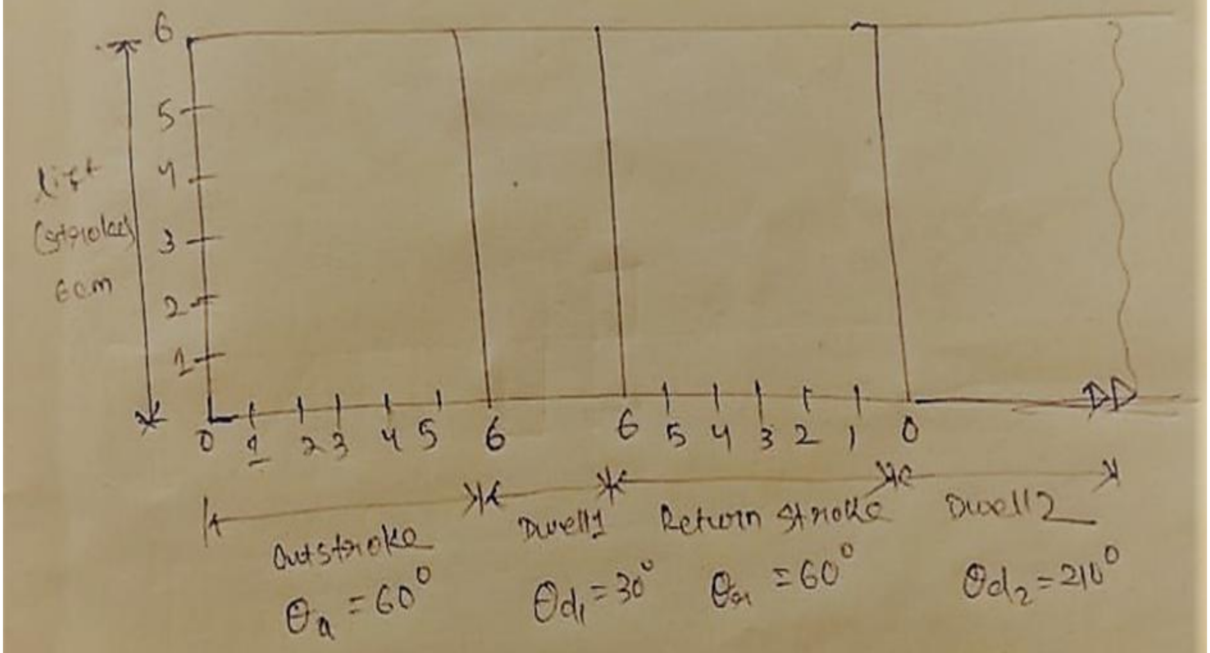
Solution:

Data given: $\theta_a = 60^0$, $\theta_{d1} = 30^0$, $\theta_r = 60^0$, $\theta_{d2} = 210^0$, lift or stroke = 6cm.

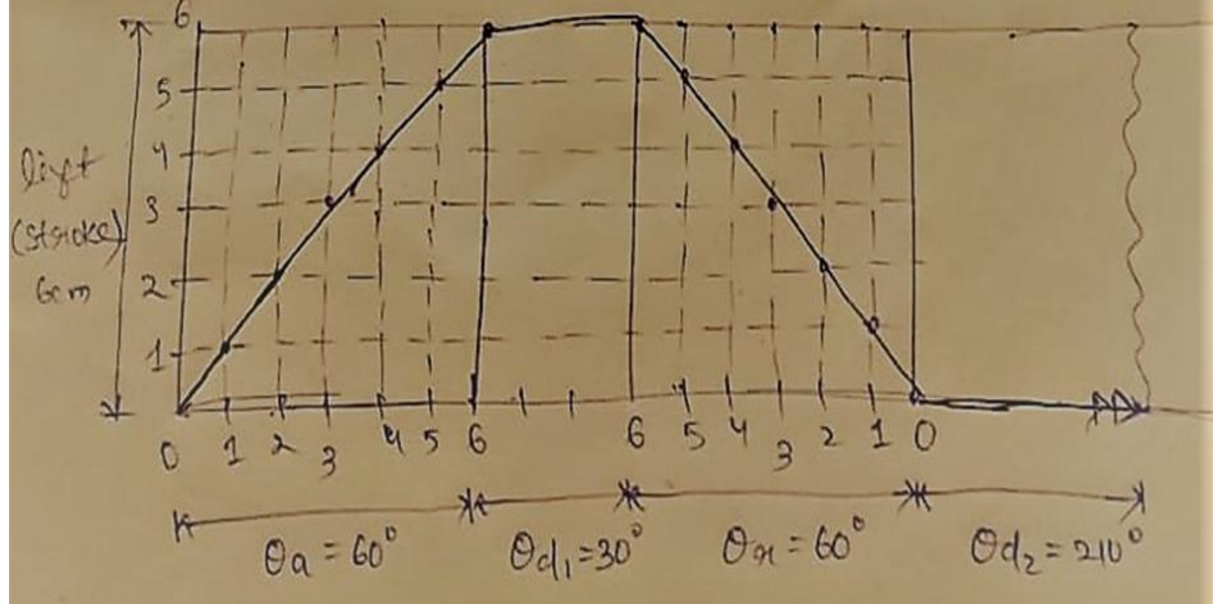
Steps:

- ⇨ Draw a vertical line and mark the lift i.e. 6cm on that line.
- ⇨ Then raw two horizontal line at the start and end of that lift.
- ⇨ Then on the horizontal bottom line mark the outstroke, dwell and return stroke by taking appropriate scale.
- ⇨ Here let's take 1cm = 10^0 on cam rotation angle nad divide the lift into six equal parts and mark them o tom6.
- ⇨ Again in outstroke mark them fom 0 to 6 and in return stroke mark them from 0 to 6.
- ⇨ Then in the outstroke mark the point (1,1) and (2,2) upto (6,6).
- ⇨ Similarly in the return stroke mark the points (1,1) upto (6,6).
- ⇨ Then connect those points and the rest angle will be the dwell angle.

step-1


6
5
4
3
2
1
lift (stroke) 6cm
0 1 2 3 4 5 6
6 5 4 3 2 1 0
Outstroke
Dwell 1
Return stroke
Dwell 2
θa = 60°
θd1 = 30°
θr = 60°
θd2 = 210°

step-2


6
5
4
3
2
1
lift (stroke) 6cm
0 1 2 3 4 5 6
6 5 4 3 2 1 0
θa = 60°
θd1 = 30°
θr = 60°
θd2 = 210°

Uniform accelerating and retardation (UARD)

- Among the types of follower movement in this type the follower accelerating in the outstroke and retard with the same movement in the return stroke.
- The construction of displacement of following diagram with cam movement in done by following steps:-

  1- Draw the cam rotation (in degrees) on x-axis and with the follower rise on y-axis (in cm).
  2- Mark the ascend or acceleration or outstroke, return stroke, from the origin.
  3- Divide the lift into equal number of parts that of the outstroke and draw horizontal and vertical line at the midpoint of the outstroke.
  4- Join the points of intersection of horizontal lines and mid-vertical line from origin up to the middle part of the vertical line and the points above the middle point are joined with the top most point of the last vertical line division.
  5- Now repeat this for the descend or retard section by taking the direction reverse to the ascend section.
  6- Then join the respective lines from the mid-vertical inclined lines with the vertical lines from each part from the outstroke and return stroke and join the locus of these points to find the curve of displacement diagram.

- Here the maximum velocity is set at the midpoint of the vertical line of outstroke and minimum at the same point in return stroke.
- **Motion Analysis** :-

Let a= uniform acceleration / retardation of follower.

Y= displacement,

We know $\frac{dv}{dt} = a$

$V = at + C_1$ (integrating both sides)

At t=0, v=0, C_1=0, hence V= at

Thus v=at

$Y = \frac{at^2}{2} + C_1 t + C_2$ (integrating with sides)

(y = the displacement)

EXAMPLE:-

Draw a follower profile raise with uniform acceleration and retardation and is lowered with simple harmonic motion:

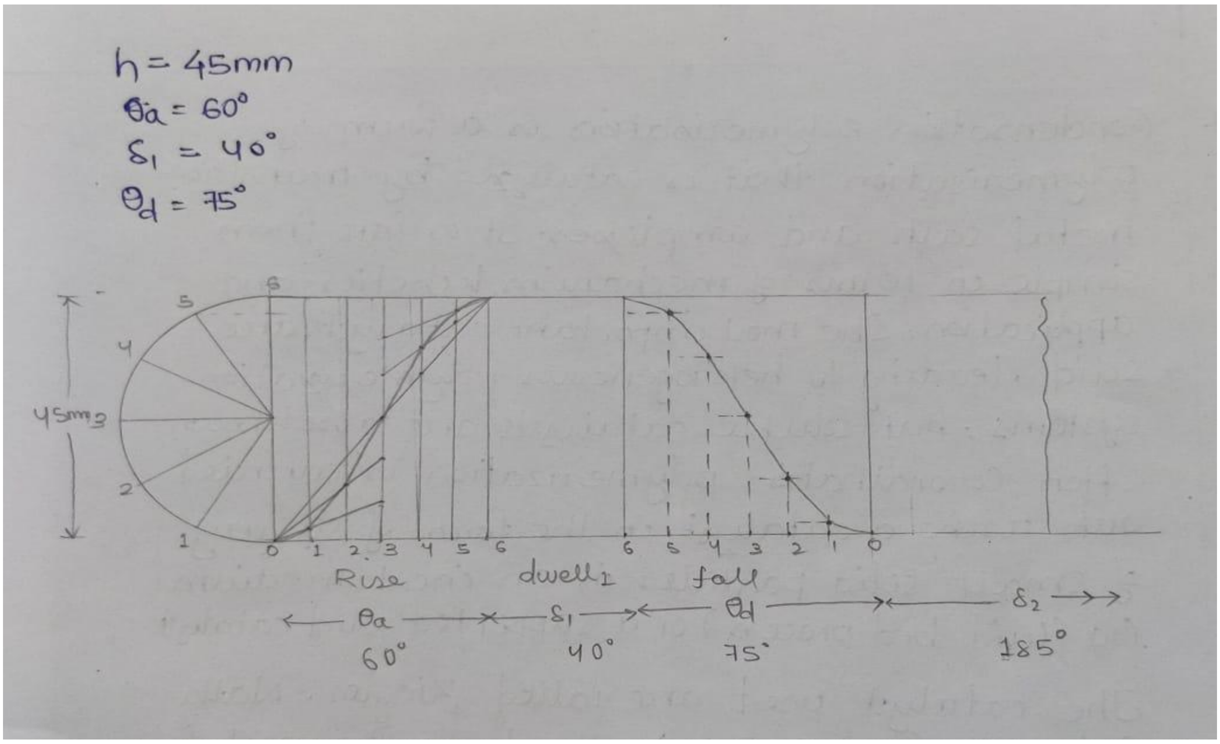
Lift of follower= 45mm

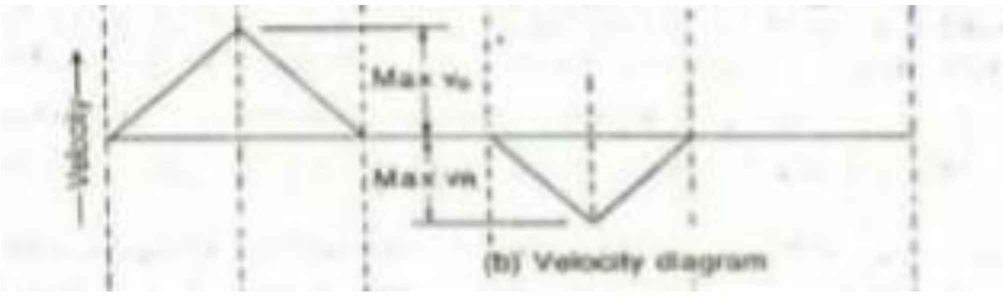
Angle of ascent= 60°

Angle of dwell= 40°

Angle of descent= 75°

Solution:





(b) Velocity diagram

GENERATION OF CAM PROFILE

INTRODUCTION:

Cam profile is the actual working surface contour of the cam. It is the contact surface knife edge, roller and flat faced follower. The fundamental law of cam design is given by Norton's, which state that the cam follower function must have contentious velocity and acceleration across the entire intake interval, thus making the jerk finite. Cam profile plays an important part in the preference of cam mechanism, so that its generation is very important.

Cam profile construction

The shape of the cam profile is obtained from the displacement of the follower. Hence the first step in construction the cam profile is to draw the displacement diagram according to the desired motion of follower. Now the actual conditions are reversed. The cam is supposed to remain fixed while the line of stroke of the follower is revolved around the cam is the opposite direction to that in which the cam actually turn on its axis. The profile of the contact surface of the follower with the cam for the successive positions of the followers gives the actual profile of the cam.

Drawing cam profile method:

I. Make the displacement diagram for the given follower motion.
II. Draw the base circle.
III. Consider the cam stationary and follower moving around it, in the direction opposite to that of the cam with reference to a vertical line from the Centre of the circle makes angle Θ_1, Θ_2, Θ_3 and Θ_4 corresponding to outstroke, dwell, instroke and dwell cam angles.
IV. Divide Θ_1 and Θ_2 into as many divisions as that on the displacement diagram.
V. From the point of intersection of the base circle and division radial lines locus corresponding to

displacement on the radial lines from the displacement diagram and joint all these points by a smooth curve which will give of the cam.

According to the cam profile drawn use of the follower in two ways:-

1) Roller follower
2) Knife edge follower

1. Roller follower

It is of 2 types

a) Radial follower
b) Offset follower

a)Radial follower

The profile of a cam radial follower has been shown in fig. The following steps may be used to draw the cam profile:

Step-1: Draw the base circle.

Step-2: Draw the follower in its 0^0 position tangent to the base circle.

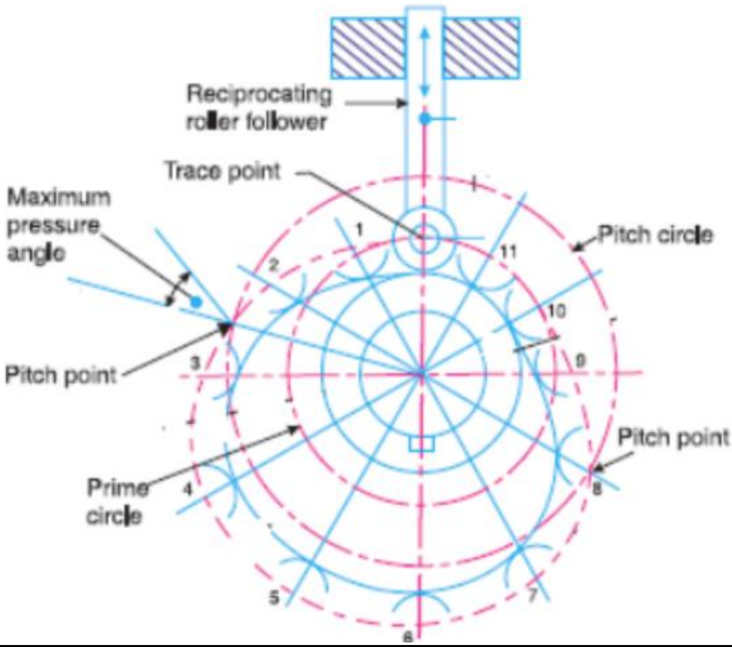
Step-3: Draw the reference circle through the Centre of the follower in its 0^0 position.

Step-4: Draw radial line from the Centre of the cam, corresponding to the vertical lines in the displacement diagram.

Step-5: Transfer displacement 1a, 2b, 3c…….. etc from the displacement diagram to the appropriate radius lines, measuring from the reference circle.

Step-6: Draw in the follower outline on the various radial lines.

Step-7: Draw a smooth curve tangent these follower outline.



Example: A disc with base circle radius of 50 mm is operating a roller follower with SHM. The lift is 25mm angle of ascent 120^0, dwell 90°, and dwell during the remaining period. The cam rotates at 300 rpm. Find the maximum velocity and acceleration during ascent. The roller radius is 10mm. Draw the ca profile when the line of reciprocation of follower passes through the cam axis.

***Solution*:**

Given: N=300 rpm, h=25mm, $\Theta_1=120^0$, $\Theta_2=90^0$

Angular velocity, ω=$\frac{2\pi\times300}{60}$= 31.416 rad/s

Maximum velocity during ascent,

v_a= $(\frac{\pi h}{2})*(\frac{\omega}{\theta 1})=(\frac{\pi\times0.025}{2})*(\frac{31.416}{\frac{\pi}{180}*120})$ = 0.5899m/s

Maximum velocity during descent,

v_d= $(\frac{\pi h}{2})*(\frac{\omega}{\theta 2})=(\frac{\pi\times0.025}{2})*(\frac{31.416}{\frac{\pi}{180}*90})$=0.785 m/s

Maximum acceleration during ascent= $(\frac{2v_a^2}{h})$= $(\frac{2(0.589)^2}{2})$= 27.75m/s^2

Maximum acceleration during descent= $(\frac{2v_d^2}{h})$= $(\frac{2(0.785)^2}{0.025})$= 49.35m/s^2

b) Offset roller follower

A cam profile with roller follower is shown in fig. The following steps may be used to draw the cam profile

Step-1: Draw the base circle.

Step-2: Draw the following in its 0^0 position tangent to the base circle.

Step-3: Draw the reference circle through the centre of the follower on its 0^0 position.

Step-4: Draw the offset circle tangent to the follower centre line.

Step-5: Divide the offset circle in a number of divisions corresponding in the displacement diagram.

Step-6: Lay off various displacements 1a, 2b, 3c ….etc along the appropriate tangent lines, measuring from the reference circle.

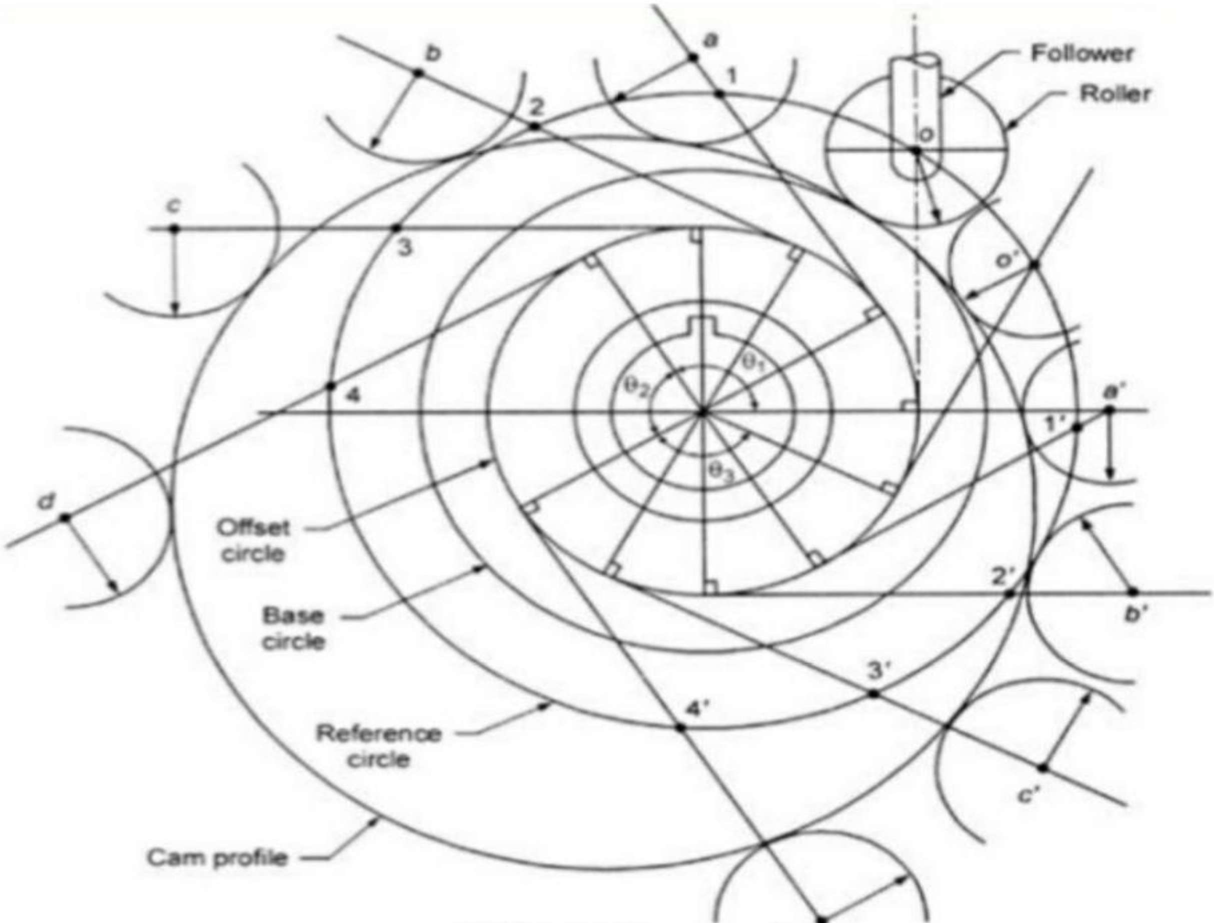
Step-7: Draw in the follower outline on the various tangent lines.

Step-8: Draw a smooth curve to these follower outlines.

Example: A cam is to give the following motion to a KNIFE-EDGED-FOLLOWER:1.Out stroke during 60° of cam rotation; 2. Dwell for the next 30° of cam rotation; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with UNIFORM VELOCITY during both the out stroke and return stroke. Draw the profile of the cam when

(a) The axis of the follower passes through the axis of the cam shaft, and

(b) The axis of the follower is offset by 20 mm from the axis of the cam shaft.



(b) Cam profile

Solution:

Given:

Outstroke angle = 60°

Dwell (outstroke) angel = 30°

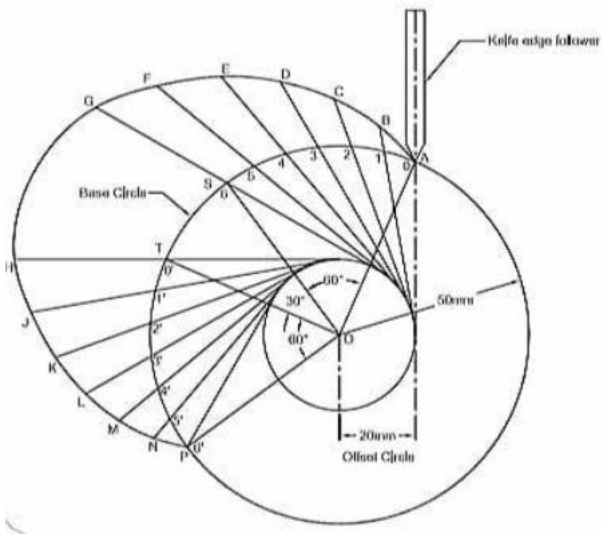
Return stroke angle = 60°

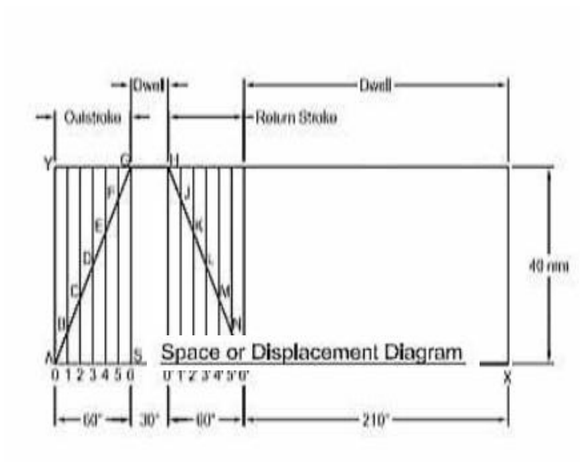
Dwell (Return stroke) angle = 210°

Stroke = 40 mm

Radius of CAM = 50mm

Offset = 20 mm





Profile of the cam when the axis of the follower Is offset by 20 mm from the axis of the cam shaft

2) Knife edge follower

The following procedure may be adopted to draw the cam profile with knife edge follower.

Step-1: Draw the displacement diagram for follower motion.

Step-2: Consider that cam remains stationary and that the follower moves round it in a direction opposite to the direction of cam rotation.

Step-3: Draw the cam base circle and divide its circumference into equal number of division depending upon the divisions used in the displacement diagram.

Step-4: Draw the various positions of follower with dotted lines corresponding to different angular displacement from the radius from which ascent is to commence.

Step-5: Draw a smooth curve tangent to the contact in different positions.

EXAMPLE:

1. Outstroke during 60°of cam rotation: 2. Dwell for the next 30° of cam rotation: 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with UNIFORM VELOCITY during both the outstroke and return strokes. Draw the profile of the cam when **(a)** the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

SOLUTION:

Given that in question

Out stroke Angle=60° (20mm)

Dwell (outstroke) angle=30° (10mm).

Return stroke Angle=60° (20mm).

Dwell (Return stroke) angle=210° (70mm)

Stroke=40mm

Radius of cam=50mm

Offset=20mm.

Assume:

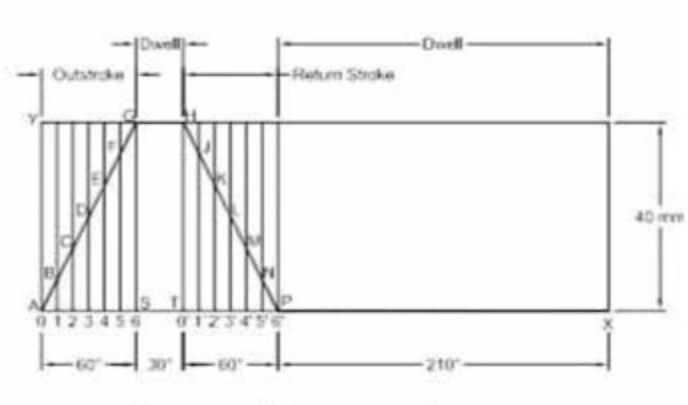
30°=10mm

360°=120mm.

Surface contact: knife edge.

Path of motion of follower: Radial

Motion of the follower



Space or Displacement Diagram

Scale-1:1