Communication Engineering Lecture Series on

Module-I

INTRODUCTION

Presented By
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Topics

1. Elements of an Electrical Communication System

   (Section 1.2 of Text book)

2. Communication Channels and their Characteristics (1.3 of Text book)

3. Mathematical Models for Communication Channels (1.4 of Text book)

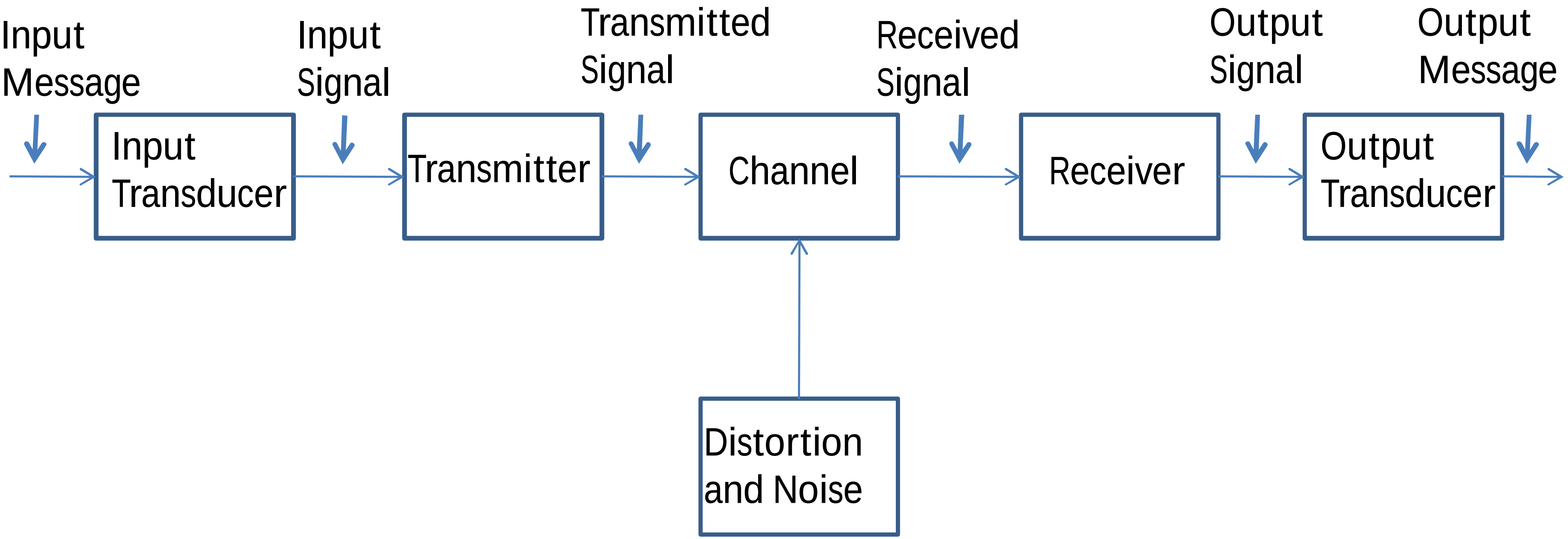
Text book-

1. John G.Proakis, M. Salehi, COMMUNICATION SYSTEMS ENGINEERING, 2nd ed. New Delhi, India: PHI Learning Private Limited, 2009

What is communication?

Communication is establishing link between two points for information exchange.

Block diagram of a communication system



The elements of a communication system

1. **Information source-**

   Generates a message that has to be transmitted. The information generated by the source may be of the form of

   - voice (speech source)
   - picture (image source)
   - text Document

2. **Input transducer-**

   Converts the output of an information source into an electrical signal that is suitable for transmission.

   eg. a microphone serves as the transducer that converts a speech signal into an electrical signal,

   a video camera converts an image into an electrical signal.

The elements of a communication system (cont...)

3. Transmitter-

- The electrical signal is converted into a form that is suitable for transmission through the physical channel or transmission medium.
- The information signal is translated in frequency domain to be transmitted into the appropriate frequency range that matches the frequency range allocated to the transmitter. For example in case of radio broadcasting each transmitter is allocated a specific frequency range so that signals transmitted by multiple radio stations do not interfere with one another.
- Transmitter performs the matching of the message signal to the channel by a process called **modulation** for information transmission.

The elements of a communication system (cont...)

Modulation is the process of varying (modulating) one or more parameters of a sinusoidal carrier signal, $V_{Carrier}(t)$ with the message signal, $V_{Message}(t)$ called the **modulating** signal.

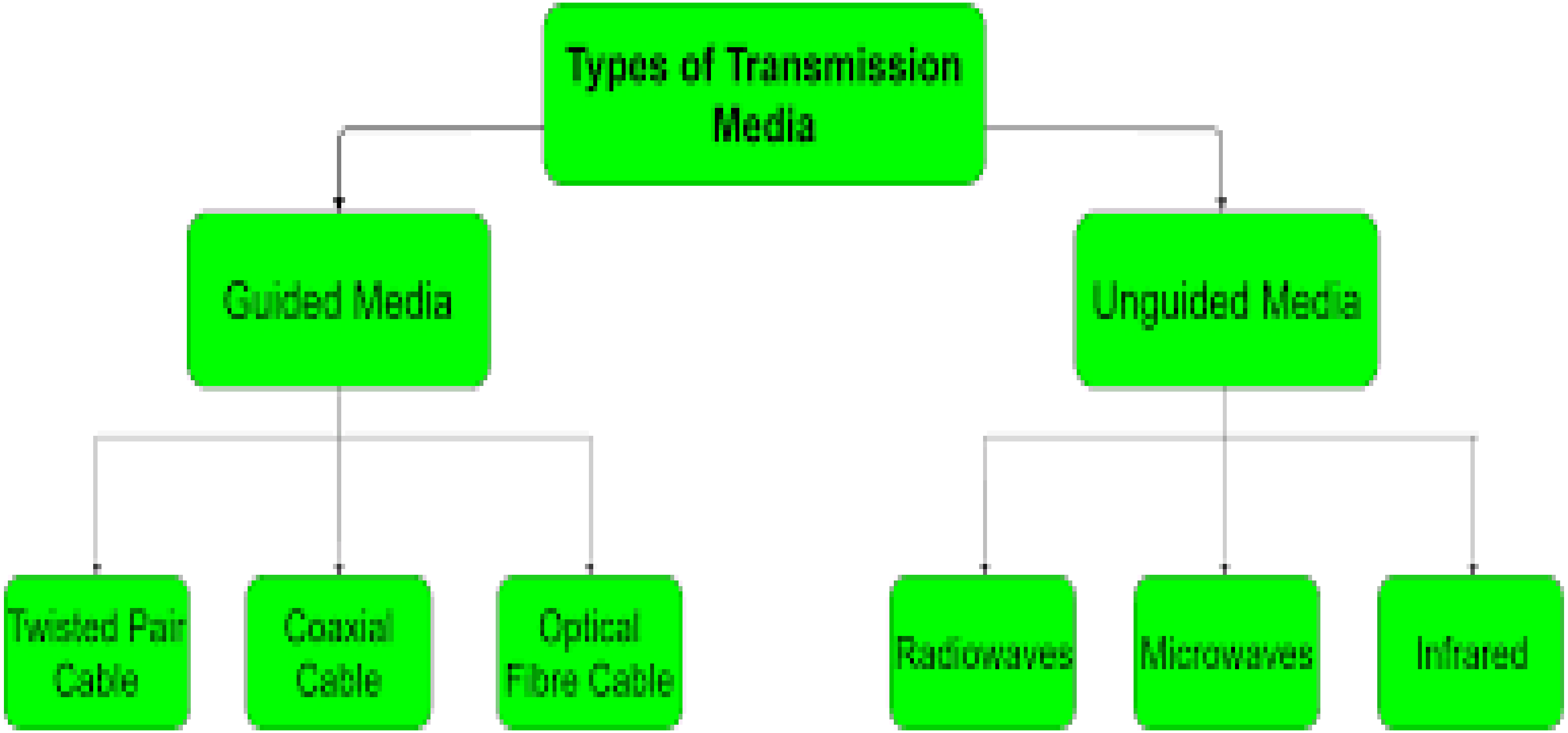
$$V_{carrier}(t) = A\cos(2\pi f_c t + \phi)$$

- Amplitude modulation -The amplitude of the carrier is varied with the message signal $V_{Message}(t)$
- Frequency modulation- The frequency of the carrier is modulated by the message signal $V_{Message}(t)$
- Phase modulation –Phase of the carrier signal is modulated by the message signal $V_{Message}(t)$.

The elements of a communication system (cont...)

4. **Channel-** The communications channel is the physical medium that is used to send the signal from the transmitter to the receiver. eg. Wired telephone channel, optical fibre cables and wireless (microwave radio).
5. **Noise-**The signal transmitted through the channel is usually corrupted in a random manner by a variety of possible mechanisms/disturbances.eg. Thermal noise, atmospheric noise, inter channel interference .The channel distorts the signal and the noise accumulates along the path.
6. **Receiver-**Recovers the message signal contained in the received signal in the presence of additive noise and possibly other signal distortion.
   - Signal demodulation- The demodulated message signal is generally degraded to some extent by the presence of these distortions in the received signal
   - Signal filtering
   - Noise suppression.

Transmission Mediums



Types of Transmission Media

Guided Media

Unguided Media

Twisted Pair Cable

Coaxial Cable

Optical Fibre Cable

Radiowaves

Microwaves

Infrared

Communication Channels and their Characteristics

1. **Wire line Channels:** Telephone network makes use of wire lines for voice signal transmission, data and video transmission.

   Guided electromagnetic channels which provide relatively modest bandwidths are

   - Twisted-pair wire lines-Have bandwidth of several hundred Kilohertz (KHz)
   - Coaxial cables -Have bandwidth of several meghertzs (MHz)

Limitations- Amplitude and phase distortions during signal transmission

Channel equalizer is used to compensate for the amplitude and phase distortions in wire line communication systems.

Communication Channels (Cont...)

2. Fibre Optic Channels:

Optical fibre cables are guided electromagnetic channels that have

- very large channel bandwidth (magnitude several order larger than coaxial cable)
- Wide array of telecommunication services, including voice, data, facsimile, and video transmission is possible.
- a relatively low signal attenuation
- reliable and secure transmission
- Highly reliable photonic devices have been developed for signal generation and signal detection.

Fibre Optic Channels (Cont...)

- The transmitter or modulator in a fibre optic communication system is a light source, either a light-emitting diode (LED) or a laser.
- Information is transmitted by varying (modulating) the intensity of the light source with the message signal. The light propagates through the fibre as a light wave and is amplified periodically (in the case of digital transmission, it is detected and regenerated by repeaters) along the transmission path to compensate for signal attenuation.
- At the receiver, the light intensity is detected by a photodiode, whose output is an electrical signal that varies in direct proportion to the power of the light impinging on the photodiode.

Communication Channels (Cont...)

Wireless Electromagnetic (EM)Channels:

- In radio communication systems, electromagnetic energy is coupled to the propagation medium by an *antenna* which serves as the radiator.
- The physical size and the configuration of the antenna depend primarily on the frequency of operation. To obtain efficient radiation of electromagnetic energy, the antenna must be longer than 1/10 of the wavelength.

For example: If a radio station transmitting in the AM frequency band, say at 1 MHz (corresponding to a wavelength $\lambda = {}^{c}/_{f} = 300\, m$ (where C is the velocity of light= $3*10^8$ m/s and f_c is carrier frequency) requires an antenna of at least 30 meters.

Electromagnetic (EM)Channels(cont..)

- Electromagnetic waves or electromagnetic radiations are synchronised oscillations of Electric and Magnetic fields. The oscillations of the two fields are perpendicular to each other and perpendicular to the direction of wave propagation, forming a transverse wave.
- Electromagnetic waves travel at the speed of light.
- The mode of propagation of electromagnetic waves in the atmosphere and in free space may be subdivided into three categories.
  - ☐ Ground wave Propagation- VLF, ELF & MF band(0.3-3 MHz)
  - ☐ Sky wave Propagation (Up to 30 MHz)
  - ☐ Space wave propagation (Line of Sight Propagation)

    (Above 30 Mhz, VHF band and higher)

Electromagnetic (EM)Channels(cont..)

Ground wave Propagation

- In the VLF and ELF, MF band (0.3-3 MHz) earth and ionosphere act as waveguide for EM wave propagation.
- In these frequency ranges, communication signals practically propagate around the globe.
- Ground wave Propagation is used to provide navigational aids from shore to ships around the world.

Ground wave propagation

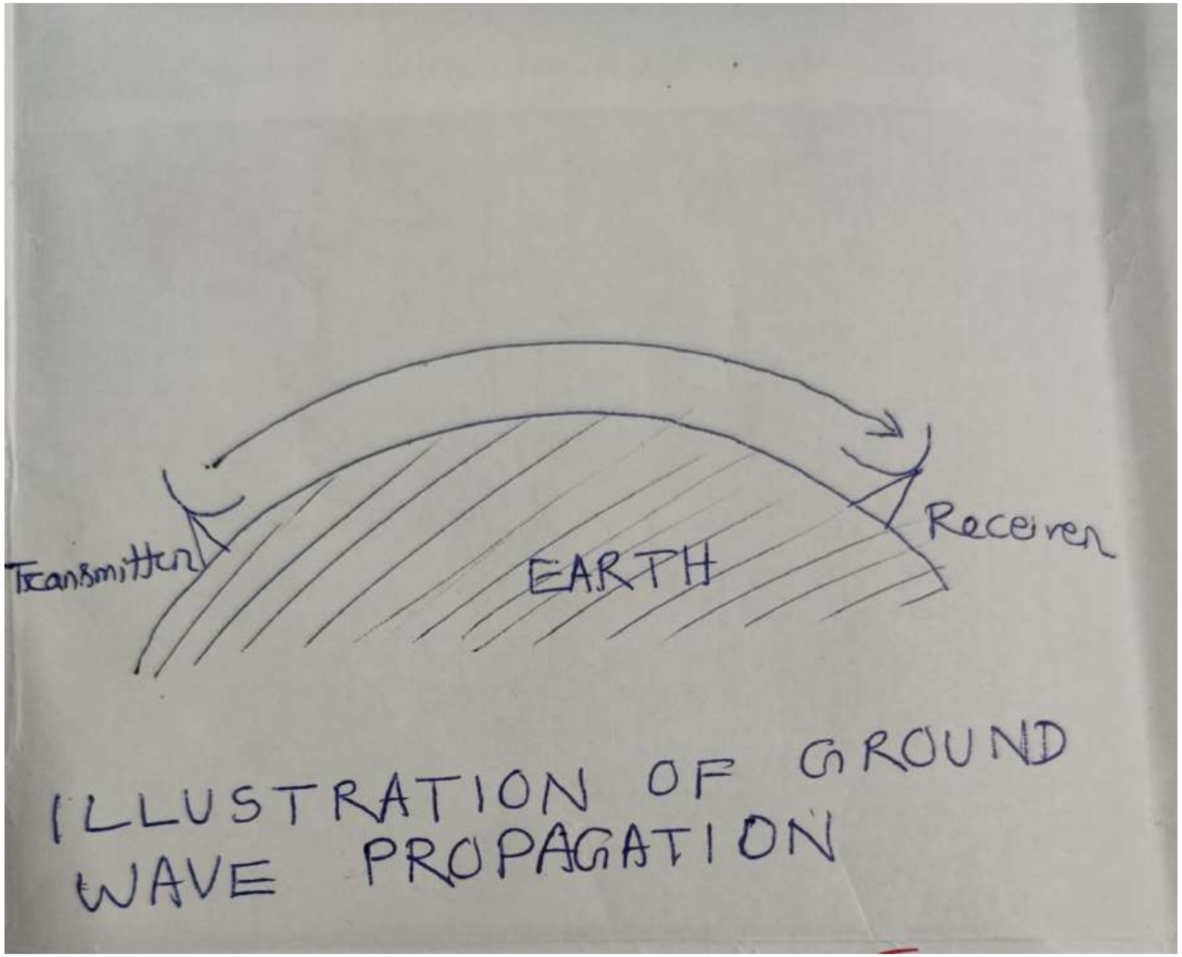


ILLUSTRATION OF GROUND WAVE PROPAGATION

Electromagnetic (EM)Channels(cont..)

Sky-wave propagation

- Sky-wave propagation results from transmitted signal being reflected (bent or refracted by total internal reflection) from the ionosphere, which consists of several layers of charged particles ranging in altitude from 30-250 miles above the surface of the earth. EM waves in HF band (3 – 30 MHz) propagate via sky wave.
- A frequently occurring problem with electromagnetic wave propagation via sky-wave in the HF frequency range is *signal multipath. Signal multipath occurs when* the transmitted signal arrives at the receiver via multiple propagation paths at different delays.
    - Signal multipath may result in intersymbol interference in a digital communication system.
    - The signal components arriving via different propagation paths may add destructively, resulting in a phenomenon called *signal fading*

Sky-wave propagation

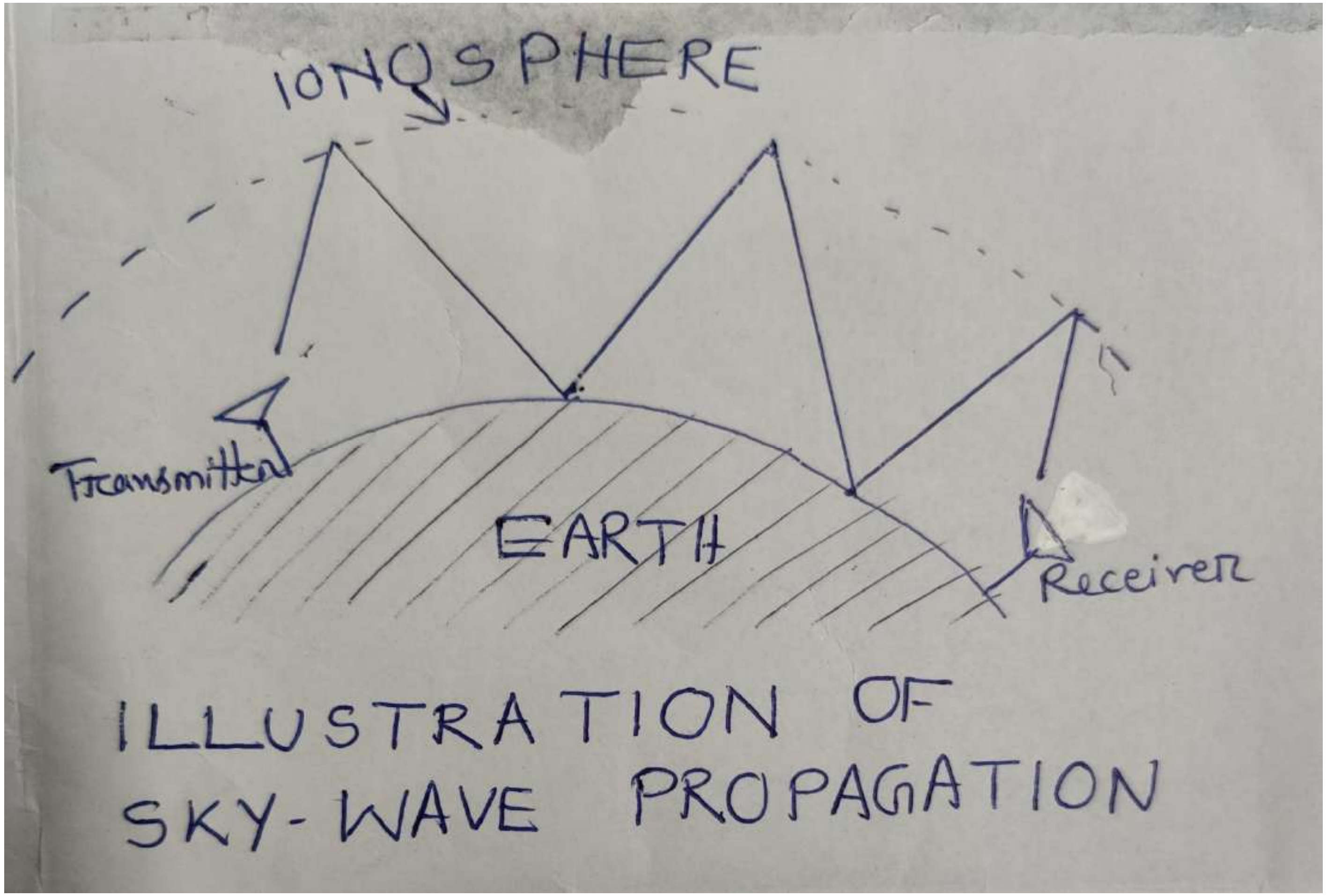


ILLUSTRATION OF SKY-WAVE PROPAGATION

Electromagnetic (EM)Channels(cont..)

Space Wave propagation

- Frequencies in the VHF band and higher (i.e. above 30 MHz), propagate through the ionosphere with relatively little loss and make satellite and extraterrestrial. communications possible.
- At frequencies in the VHF band and higher, the dominant mode of electromagnetic propagation is Space Wave propagation, also known as line-of-sight (LOS) propagation. For terrestrial communication systems, this means that the transmitter and receiver antennas must be in direct LOS with relatively little or no obstruction. For this reason television stations transmitting in the VHF and UHF frequency bands mount their antennas on high towers in order to achieve a broad coverage area.
- The coverage area for LOS propagation is limited by the curvature of the earth. If the transmitting antenna is mounted at a height h *feet above the surface* of the earth, the distance to the radio horizon, assuming no physical obstructions such as mountains, is approximately $d = \sqrt{2h}$ *miles*

Line-of-sight (LOS) propagation

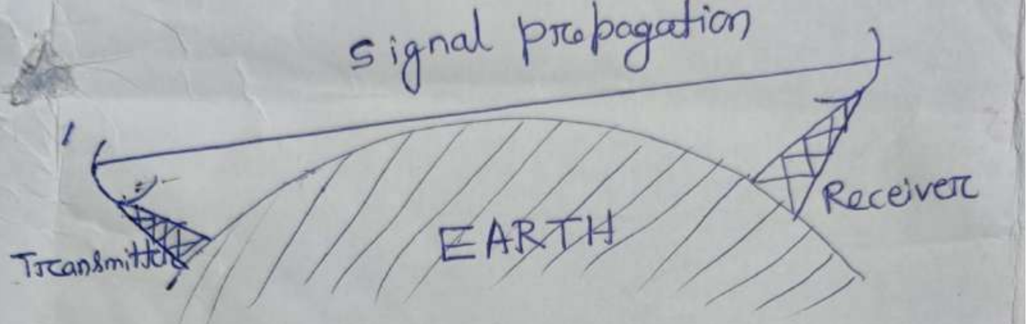
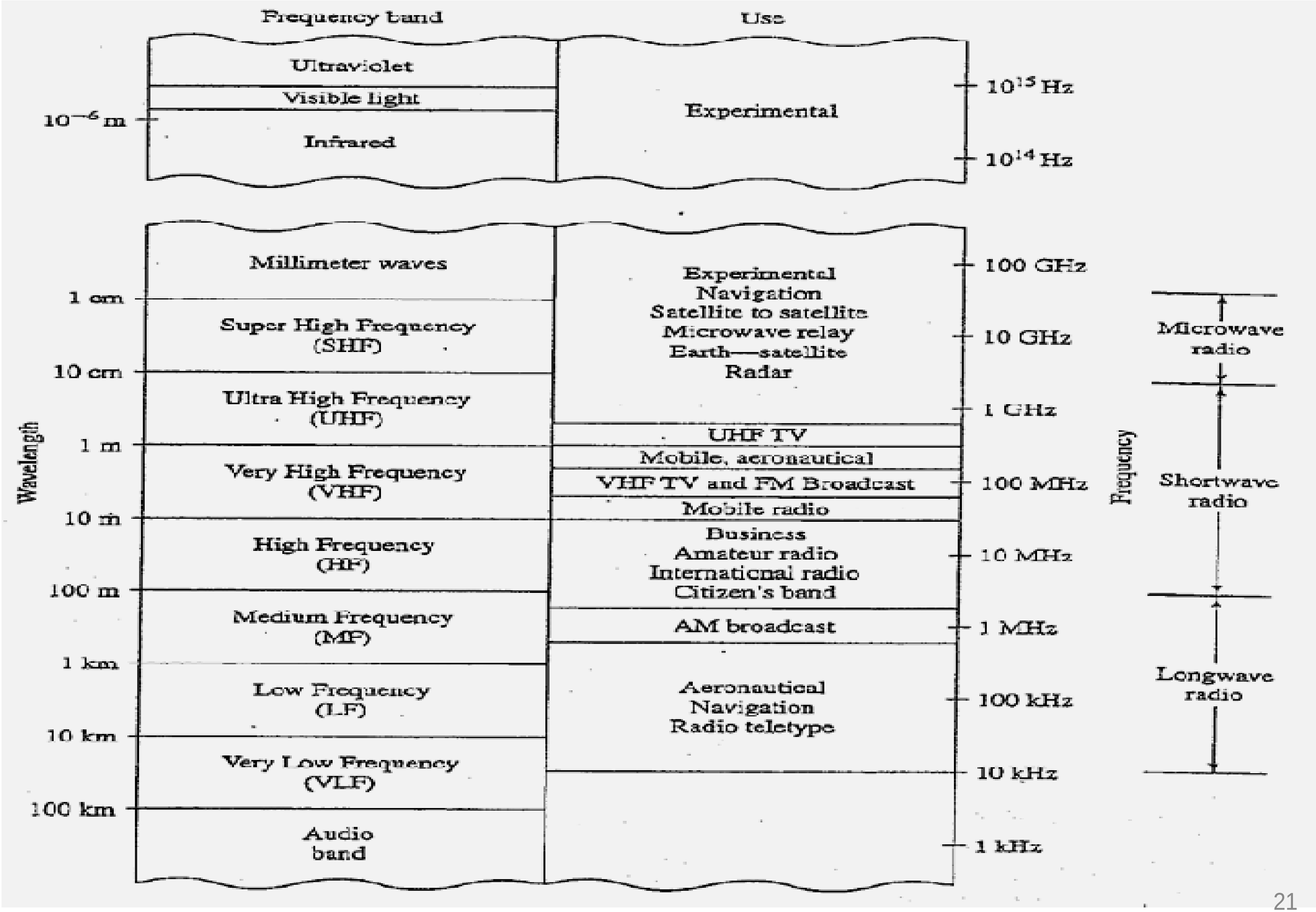


ILLUSTRATION OF LINE OF SIGHT PROPAGATION

Frequency range for wireless EMchannel



Communication Channels (Cont...)

Underwater Acoustic Channels:

- Electromagnetic waves do not propagate over long distances underwater, except at extremely low frequencies.

However, the transmission of signals at such low frequencies is prohibitively expensive because of the large and powerful transmitters required.

- The attenuation of electromagnetic waves in water can be expressed in terms of the skin depth, which is the distance at which the signal is attenuated by $1/e$.
- For sea water, the skin depth is $\delta = 250/\sqrt{f}$ where f is expressed in Hz and δ is in meters.
- Sensors are placed underwater below the surface of the ocean from C where it is possible to relay the data via a satellite to a data collection centre.
- A shallow water acoustic channel is characterized as a multipath channel due to signal reflection from the surface and bottom of the sea.
- Due to wave propagation **signal multipath** components undergo time varying propagation delays which result in signal fading.

Communication Channels (Cont...)

- **Storage Channel:**
- Magnetic tape (digital audio tape and video tape, magnetic disks) used for storing large amounts of computer data, and optical disks used for computer data storage, music (compact disks), and video are examples of data storage systems that can be characterized as communication channels.
- The process of storing data on a magnetic tape or a magnetic or optical disk is equivalent to transmitting a signal over a telephone or a radio channel.
- The readback process and the signal processing involved in storage systems to recover the stored information is equivalent to the functions performed by a receiver in a telephone or radio communication system to recover the transmitted information.
- Additive noise generated by the electronic components and interference from adjacent tracks is generally present in the readback signal of a storage system, just as is the case in a telephone or a radio communication system.

Mathematical Models for Communication Channels

Mathematical models for Communication Channels reflect the most important characteristics of the transmission medium. The mathematical model for the channel is used in the design of the channel encoder and modulator at the transmitter and the demodulator and channel decoder at the receiver.

Channel models to characterize many physical communication channels that we encounter in practice are:

- **The Additive noise channel**
- **The Linear Time-invariant Filter Channel with Additive noise**
- **The Linear Time-Variant Filter Channel with Additive noise**

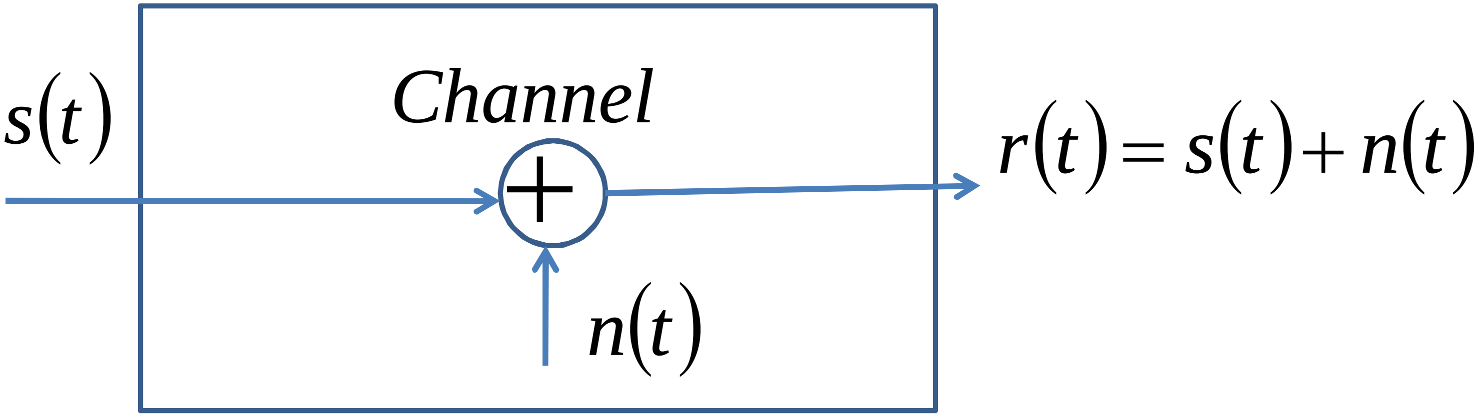
The Additive Noise Channel

- Simplest mathematical model of communication channel
- The transmitted signal $s(t)$ is corrupted by an additive random noise process $n(t)$.

  Physically, the additive noise process may arise from

  - electronic components and amplifiers at the receiver of the communication system
  - interference encountered in transmission, as in the case of radio signal transmission.
- If the noise is introduced primarily by electronic components and amplifiers at the receiver, it may be characterized as thermal noise. This type of noise is characterized statistically as a *Gaussian noise process and* the resulting mathematical model for the channel is usually called the additive Gaussian noise channel

The Additive noise channel



Channel attenuation is easily incorporated into the model. When the signal undergoes attenuation in transmission through the channel, the received signal is

$$r(t)=as(t)+n(t)$$

where *a* represents the attenuation factor.

The Linear Time-invariant Filter Channel with Additive Noise

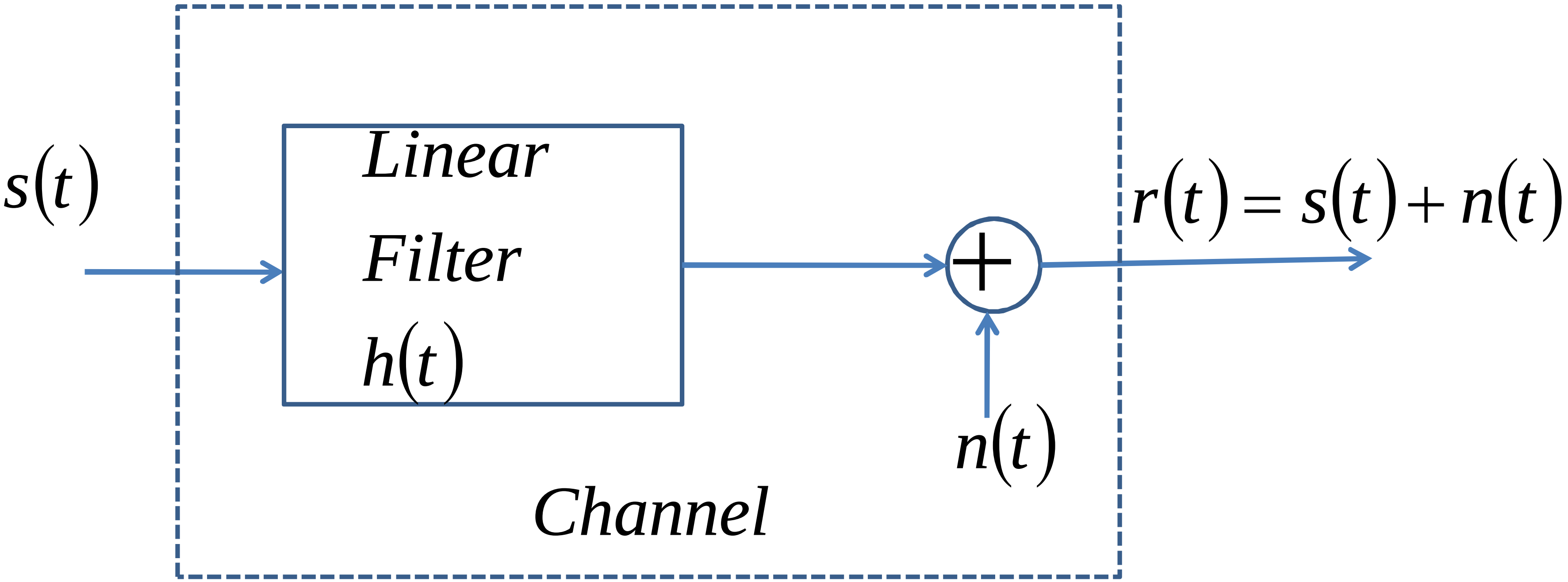
In some physical channels such as wire line telephone channels, filters are used to ensure that the transmitted signals do not exceed specified bandwidth limitations and, thus, do not interfere with one another.

Such channels are generally characterized mathematically as linear filter channel with additive noise. If the channel input is the signal $s(t)$, then the channel output is the signal

$$\begin{aligned} r(t) &= s(t) * h(t) + n(t) \\ &= \int_{-\infty}^{+\infty} h(\tau) s(t-\tau) d\tau + n(t) \end{aligned}$$

where $h(t)$ is the impulse response of the filter and $n(t)$ is the noise and $*$ denotes convolution.

The Linear Time-invariant Filter Channel with Additive noise



$s(t)$

Linear

Filter

$h(t)$

$r(t) = s(t) + n(t)$

$n(t)$

Channel

The Linear Time-Variant Filter Channel

- Physical channels such as underwater acoustic channels and ionospheric radio channels which result in time-variant multipath propagation of the transmitted signal may be characterized mathematically as time-variant linear filters.

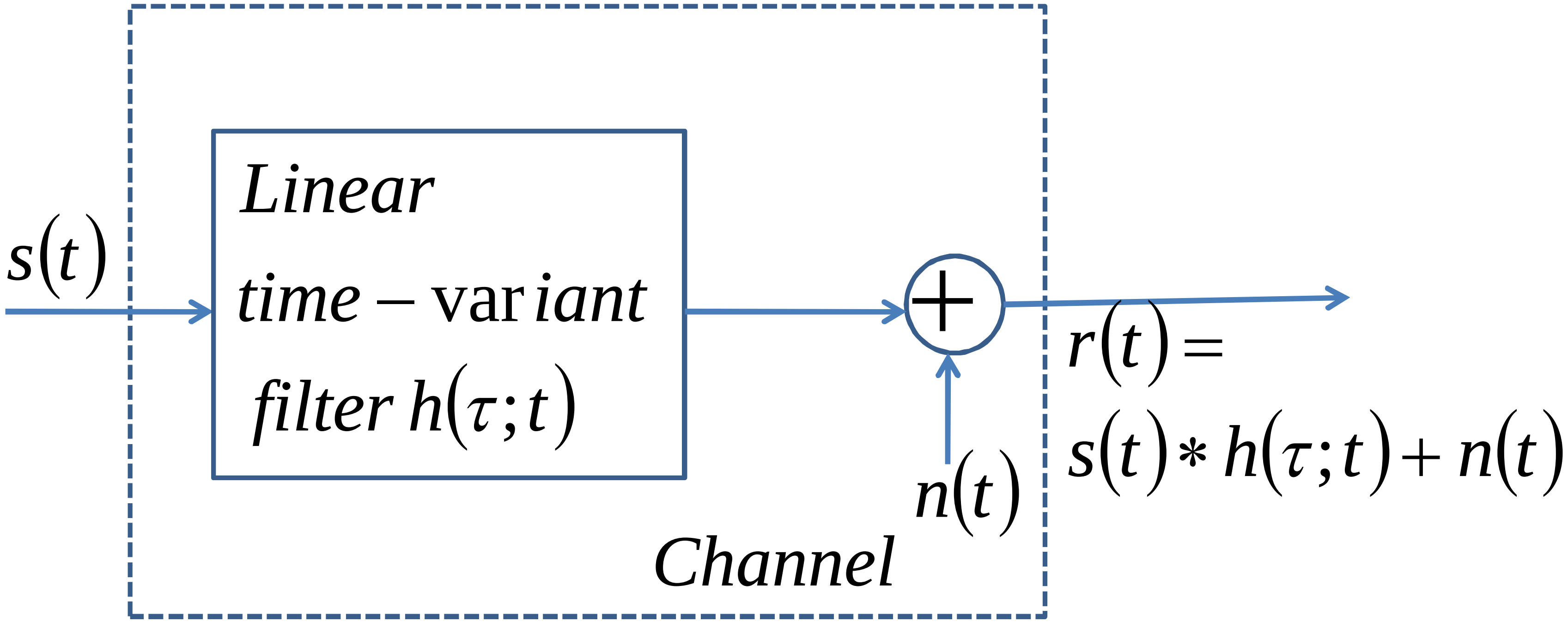
  Such channels are characterized by time-variant impulse response $h(\tau;t)$

  where $h(\tau;t)$ is the impulse response at time *t*, due to impulse applied at time $t-\tau$ and

  τ represents the "age" or elapsed time variable.
- For a linear time variant filter channel with additive noise $n(t)$ and input signal $s(t)$ the output signal is

$$
\begin{aligned}
r(t) &= s(t) * h(\tau;t) + n(t) \\
&= \int_{-\infty}^{+\infty} h(\tau;t)s(t-\tau)d\tau + n(t)
\end{aligned}
$$

The Linear Time-Variant Filter Channel



$s(t)$

Linear time-variant filter $h(\tau;t)$

$n(t)$

Channel

$$r(t) = s(t) * h(\tau;t) + n(t)$$

The Linear Time-Variant Filter Channel

A good model for multipath signal propagation through physical channels such as underwater acoustic channels, ionosphere (at frequencies below 30 MHz) and mobile cellular radio channels has time variant impulse response of the form

$$h(\tau;t)=\sum_{k=1}^{L}a_k(t)\delta(t-\tau_k)$$

where $\{a_k(t)\}$ represents the possibly time-variant attenuation factors for L propagation paths. The received signal has the form

$$r(t)=\sum_{k=1}^{L}a_k(t)s(t-\tau_k)+n(t)$$

Hence the received signal consists of L multipath components where each component is attenuated by $\{a_k(t)\}$ and delayed by $\{\tau_k\}$.

References

1. John G.Proakis, M. Salehi, COMMUNICATION SYSTEMS ENGINEERING, 2nd ed. New Delhi, India: PHI Learning Private Limited, 2009

Thank you!

Any queries?

COMMUNICATION ENGINEERING LECTURE SERIES
MODULE II
PART I

PRESENTED BY:

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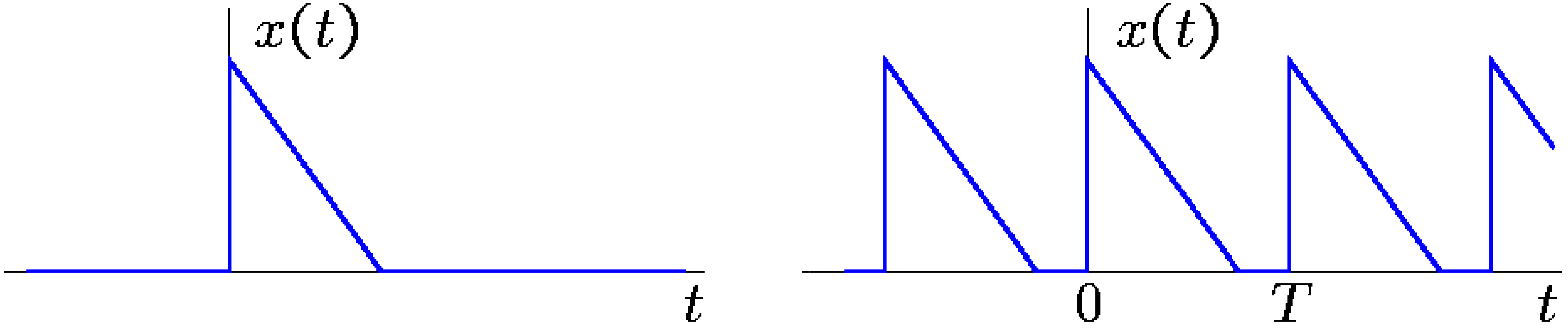
ASST. PROF., DEPT. OF EE, IGIT SARANG, ODISHA

CLASSIFICATION OF SIGNALS

I. PERIODIC AND A-PERIODIC SIGNALS

Periodic signals are such that $x(t+T) = x(t)$ for all t. The smallest value of T that satisfies the definition is called the *period*. Below on the left below is an aperiodic signal, with a periodic signal shown on the right.



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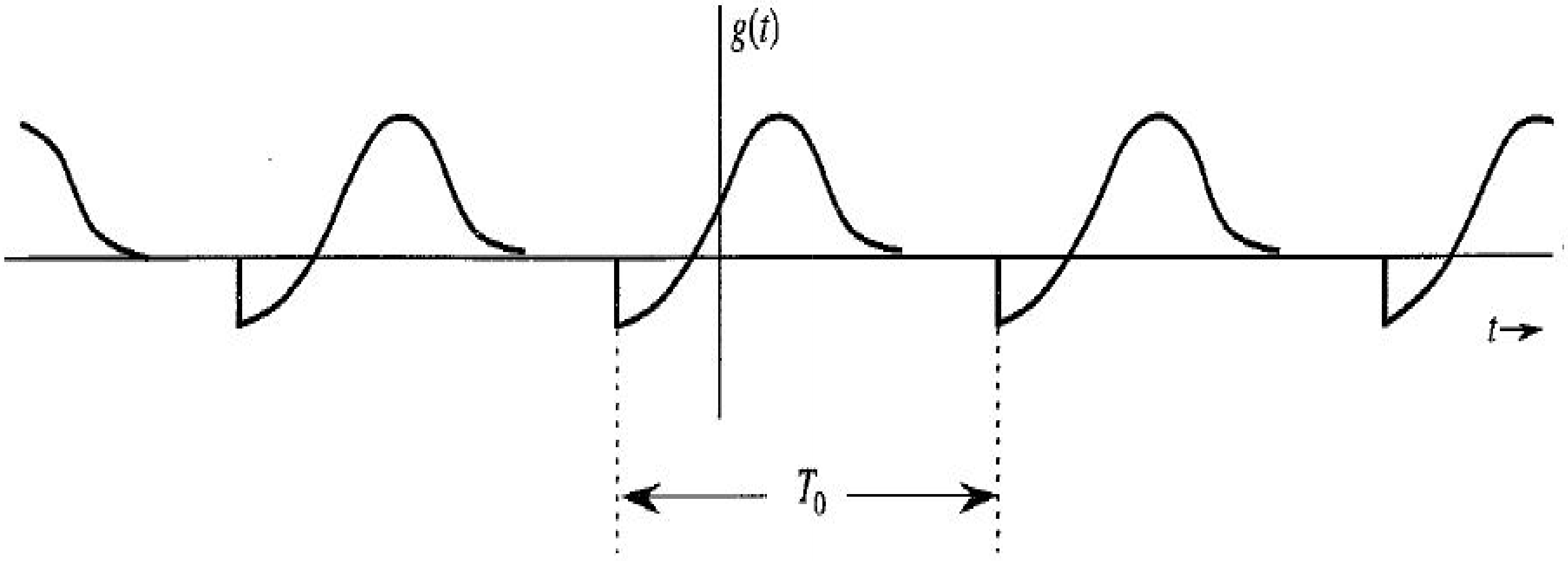


PERIODIC SIGNAL WITH PERIOD T_O

II. CONTINUOUS TIME AND DISCRETE TIME SIGNALS

A signal that is specified for every value of time is a **continuous time signal**.

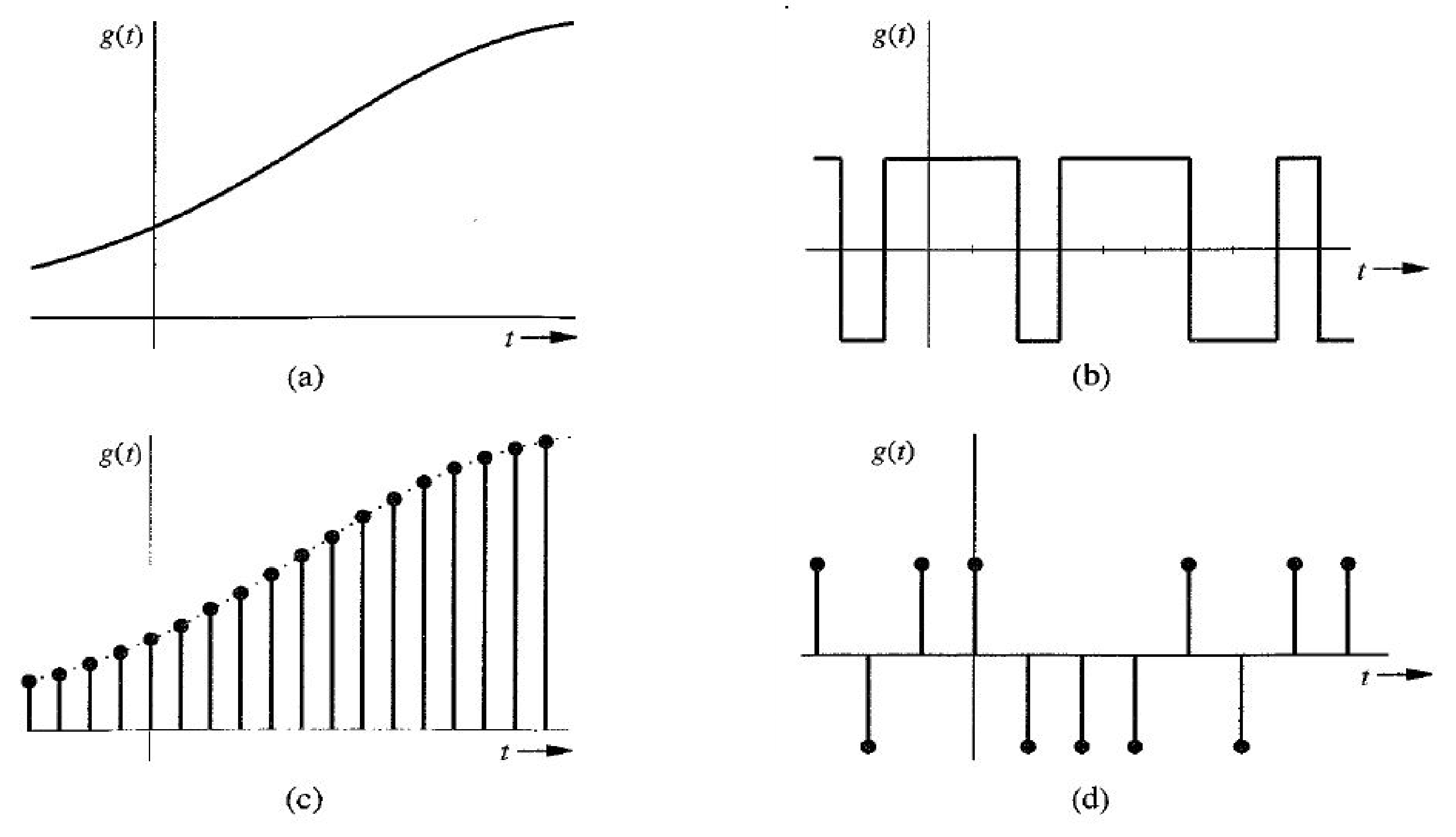
The signals that are specified only at discrete values of time are called **discrete time signals**.

III. ANALOG AND DIGITAL SIGNALS

A signal whose amplitude can take on any value in a continuous range is an **analog signal**.

A signal whose amplitude can take only a finite number of values is a digital signal for example a **binary signal**.

CONTD…



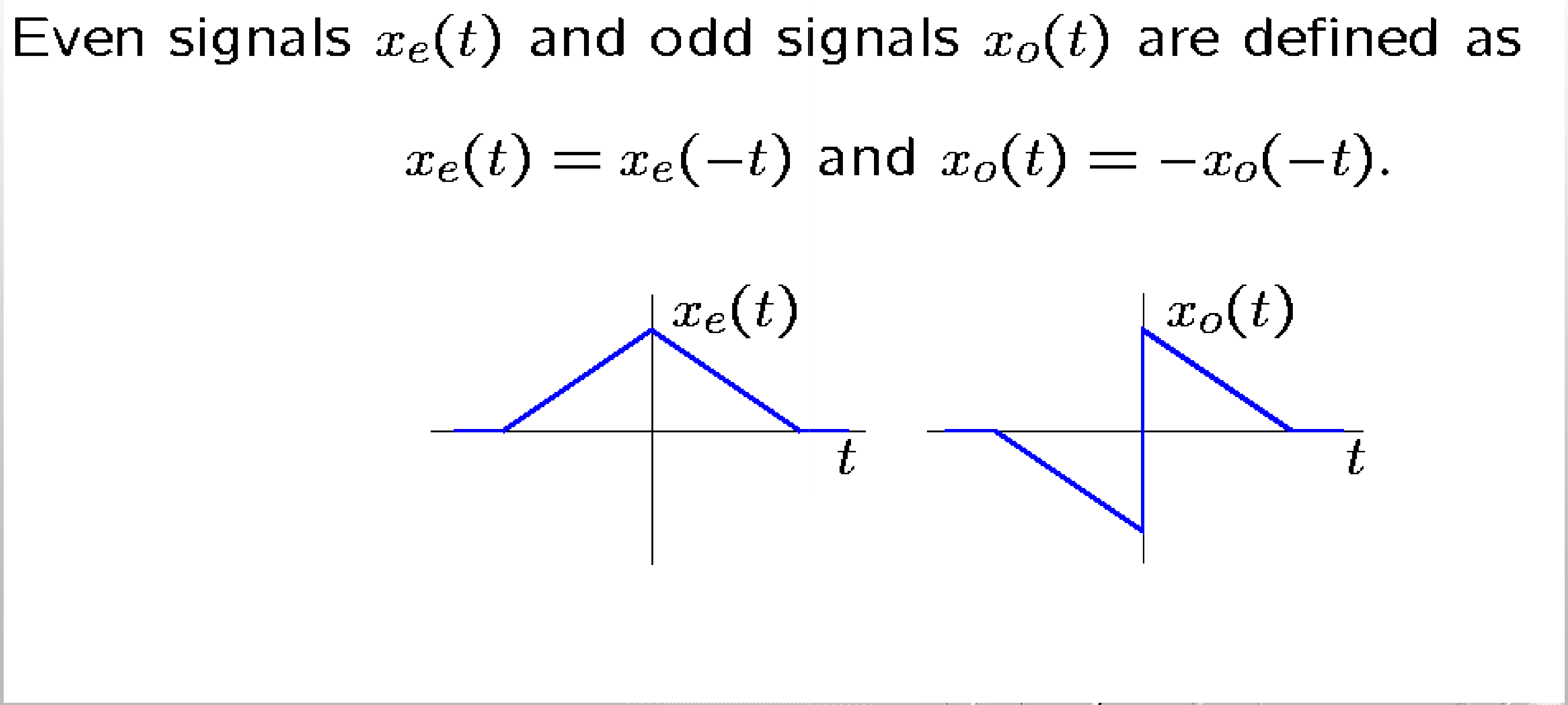
(a) (b)

(c) (d)

(a) Analog continuous time, (b) Digital continuous time, (c) Analog discrete time, (d) Digital discrete time

IV. EVEN AND ODD SIGNALS

Even signal exhibits symmetry in time domain and odd signal exhibits anti-symmetry.



Even signals $x_e(t)$ and odd signals $x_o(t)$ are defined as

$$x_e(t) = x_e(-t) \text{ and } x_o(t) = -x_o(-t).$$

V. ENERGY AND POWER SIGNAL

A signal with finite energy is a energy signal. For a signal *g(t)* its energy is calculates by:

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt < \infty$$

A signal with finite power is a power signal. For a signal *g(t)* its power is calculates by:

$$0 < \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt < \infty$$

- **A signal with finite energy has zero power and signal with finite power has infinite energy**

VI. DETERMINISTIC AND RANDOM SIGNALS

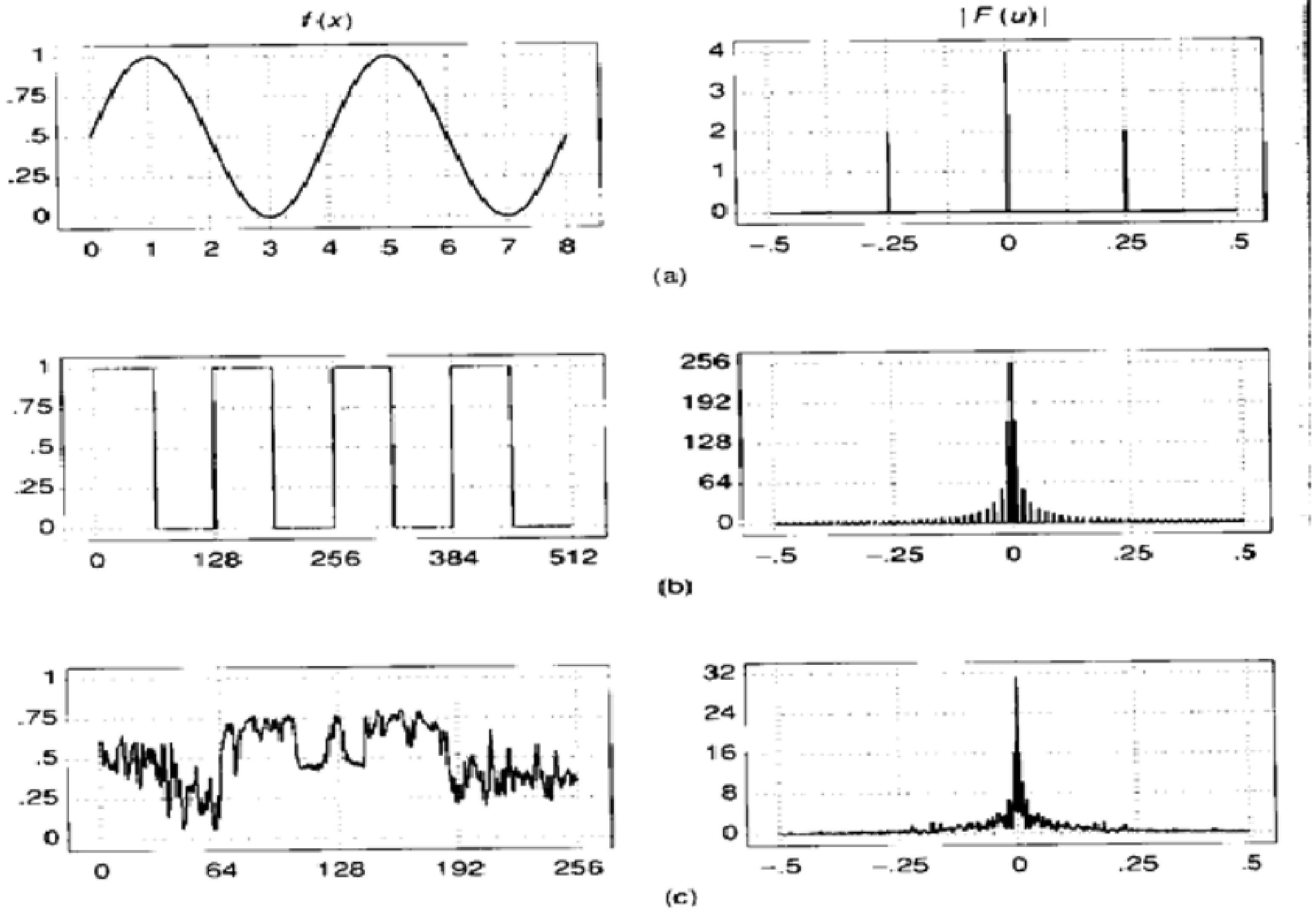
- A signal whose physical description is known completely, in either a mathematical form or graphical form is a deterministic signal.
- If a signal is only defined by its probabilistic description such as mean value, mean square value and so on rather than its mathematical form or graphical form is a random signal. Most of the noise signals are random signals

SIGNAL REPRESENTATION

1. Time domain representation
2. Frequency domain representation

- In time domain representation a signal is a time varying quantity. In frequency domain representation a signal is represented by its frequency spectrum.
- To obtain frequency spectrum of a signal Fourier series and Fourier transform are used.

REPRESENTATION OF SIGNAL IN TIME DOMAIN AND FREQUENCY DOMAIN



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1. JOHN G. PROAKIS, M. SALEHI, COMMUNICATION SYSTEMS ENGINEERING, 2ND ED. NEW DELHI, INDIA: PHI LEARNING PRIVATE LIMITED, 2009.
2. MODERN DIGITAL AND ANALOG COMMUNICATION SYSTEMS, BY B.P. LATHI, OXFORD.
3. SANJAY SHARMA, COMMUNICATION SYSTEMS (ANALOG AND DIGITAL), S. K. KATARIA & SONS PUBLISHERS.

THANKYOU !

ANY QUERIES?

COMMUNICATION ENGINEERING LECTURE SERIES
MODULE II
PART II (A)

PRESENTED BY:

DR. (MRS.) ADITI CHATTERJEE

ASST. PROF., DEPT. OF EE, IGIT SARANG, ODISHA

FOURIER SERIES

- **WHY DO WE NEED FOURIER ANALYSIS?**

In communication we send and receive information laced signal over a medium, the medium and the hardware corrupts the signal. The receiver has to extract the information from the corrupted signal. The transmitted signal have well defined spectral contents, so if the receiver can do spectral analysis of the received signal then it can extract the information.

- **WHY FOURIER SERIES IS REQUIRED?**

To represent any periodic power signal as infinite sum of sines and cosines at different frequencies.

TRIGONOMETRIC FOURIER SERIES

function $f(t)$ is periodic if it is defined for all real t and there is some positive number, T such that ,

$$x(t+T)=x(t)$$

he function can be represented by a trigonometric series as:

$$f(t)=a_0+a_1\cos\omega t+a_2\cos 2\omega t+\ldots\ldots+a_n\cos n\omega t$$

$$+b_1\sin\omega t+b_2\sin 2\omega t+\ldots.b_n\sin n\omega t$$

$$f(t)=a_0+\sum_{n=1}^{\infty}a_n\cos n\omega t+\sum_{n=1}^{\infty}b_n\sin n\omega t$$

- **WE WANT TO DETERMINE THE FOURIER COEFFICIENTS**

a_0 **is the average (dc) value of the function,** $f(t)$ **.**

given by:

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt \qquad n = 1, 2, \cdots$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt \qquad n = 1, 2, \cdots$$

Where, T is the time period

SYMMETRY IN FOURIER SERIES

EVEN FUNCTIONS SYMMETRY: $f(-t) = f(t)$

- Even functions can solely be represented by cosine waves because, cosine waves are even functions.

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega t dt, \quad b_n = 0$$

ODD FUNCTION SYMMETRY: $f(-t) = -f(t)$

- Odd functions can solely be represented by sine waves because, sine waves are odd functions.

The Fourier series of an odd function is expressed in terms of a sine series.

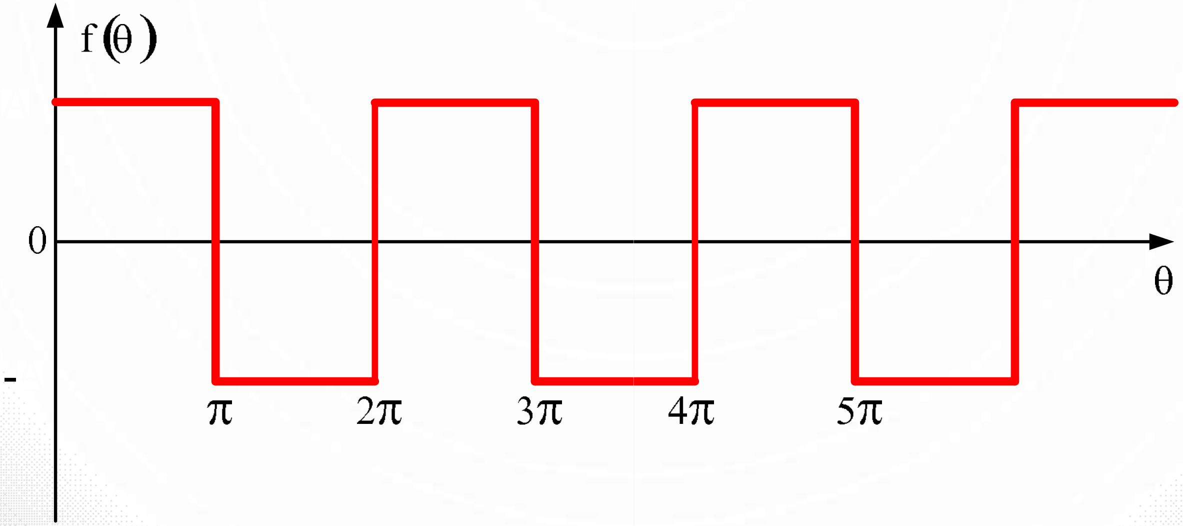
$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$a_0 = 0, a_n = 0,$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\,dt$$

SOME SOLVED EXAMPLES

Ex. 1. Find the Fourier series of the following periodic function.



$$f(\theta) = A \quad when \quad 0 < \theta < \pi$$

$$= -A \quad when \quad \pi < \theta < 2\pi$$

$$f(\theta + 2\pi) = f(\theta)$$

SOLUTION

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} f(\theta)d\theta + \int_{\pi}^{2\pi} f(\theta)d\theta\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} A\,d\theta + \int_{\pi}^{2\pi} -A\,d\theta\right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta\, d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} A\cos n\theta\, d\theta + \int_{\pi}^{2\pi} (-A)\cos n\theta\, d\theta\right]$$

$$= \frac{1}{\pi}\left[A\frac{\sin n\theta}{n}\right]_0^{\pi} + \frac{1}{\pi}\left[-A\frac{\sin n\theta}{n}\right]_{\pi}^{2\pi} = 0$$

CONTD….

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta\, d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} A\sin n\theta\, d\theta + \int_{\pi}^{2\pi}(-A)\sin n\theta\, d\theta\right]$$

$$= \frac{1}{\pi}\left[-A\frac{\cos n\theta}{n}\right]_0^{\pi} + \frac{1}{\pi}\left[A\frac{\cos n\theta}{n}\right]_{\pi}^{2\pi}$$

$$= \frac{A}{n\pi}\left[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right]$$

$$b_n = \frac{A}{n\pi}\left[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right]$$

$$= \frac{A}{n\pi}\left[1+1+1+1\right]$$

$$= \frac{4A}{n\pi} \quad \text{when n is odd}$$

CONTD…

$$b_n = \frac{A}{n\pi}\left[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right]$$

$$= \frac{A}{n\pi}\left[-1+1+1-1\right]$$

$$= 0 \quad \text{when n is even}$$

Therefore, the corresponding Fourier series is

$$\frac{4A}{\pi}\left(\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \frac{1}{7}\sin 7\theta + \cdots\right)$$

REFERENCES

1. JOHN G. PROAKIS, M. SALEHI, COMMUNICATION SYSTEMS ENGINEERING, 2ND ED. NEW DELHI, INDIA: PHI LEARNING PRIVATE LIMITED, 2009.
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ANY QUERIES?

Communication Engineering Lecture Series
on
Module-II
Frequency Domain Analysis of Signals and Systems
Part II (B)

Presented By
Dr Pranati Das
Asso. Prof., Electrical Engg. Department
IGIT Sarang, Odisha

Topics

1. Fourier Series

(Section 2.1 and 2.1.1 of Text book)

Text book-

1. John G.Proakis, M. Salehi, COMMUNICATION SYSTEMS ENGINEERING, 2nd ed. New Delhi, India: PHI Learning Private Limited, 2009

Frequency Domain Analysis of Signals and System

The analysis of signals and linear systems in the frequency domain is based on representation of the signals in terms of the frequency variable and this is done through employing following representations:

- Fourier series- It is applied to periodic signals
- Fourier transform- It can be applied to periodic and nonperiodic signals.

Dirichlet conditions:

Given a periodic signal $x(t)$ with period T_0

- $x(t)$ is absolutely integrable over its period; i.e.

$$\int_{T_0} |x(t)| dt < \infty$$

- $x(t)$ has finite number of maxima and minima within each period.
- $x(t)$ has a finite number of discontinuities within each period. At point of discontinuity

$$\pm x(t) = \begin{cases} x(t)\, if\ \ x(t)\, is\, continuous\, at\, t \\ \dfrac{x(t+)+x(t-)}{2}\, if\ \ x(t)\, is\, discontinuous\, at\, t \end{cases}$$

If Dirichlet conditions are satisfied then $x(t)$ can have Fourier series representation; i.e. the periodic signal $x(t)$ can be represented as linear combinations of harmonically related complex exponentials.

Fourier series representation of a continuous time (CT) periodic signal

A periodic signal $x(t)$ with period T_0 and satisfying Dirichlet conditions can have Fourier series expansion.

Synthesis equation: $x_{\pm}\left(t\right) = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi\left(\frac{n}{T_0}\right)t} \qquad (1)$

Analysis equation: $x_n = \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)e^{-j2\pi\frac{n}{T_0}t}\,dt \qquad (2)$

This pair of equations (1) and (2) define Fourier series representation of a periodic CT signal. The set of coefficients $\left\{x_n\right\}$ are often called the Fourier series coefficients or the spectral coefficients of $x(t)$. These complex coefficients measure the portion of the signal $x(t)$ at each harmonic of the fundamental frequency $f_0 = \frac{1}{T_0}$.

Fourier series (cont...)

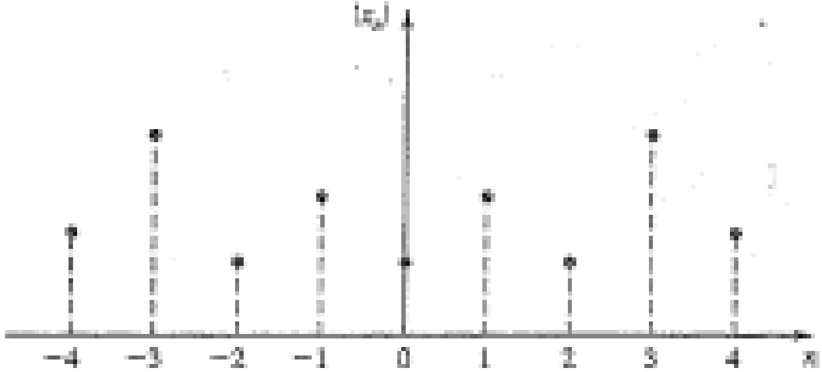
- The coefficients $\{x_n\}$ are called the Fourier series coefficients of the signal $x(t)$. These are, in general, complex numbers.
- The parameter α *in the limits of the integral is arbitrary. It can be chosen to* simplify computation of the integral. Usually $\alpha = 0$ *or* $\alpha = -T_0/2$ *are good* choices.
- The quantity $f_0 = \frac{1}{T_0}$ is called the *fundamental frequency* of the signal $x(t)$.
- The frequencies of the complex exponential signals are n^{th} multiples of this fundamental frequency. The multiple of the fundamental frequency (for positive n *'s)* is called the n^{th} *harmonic.*

Fourier series (cont...)

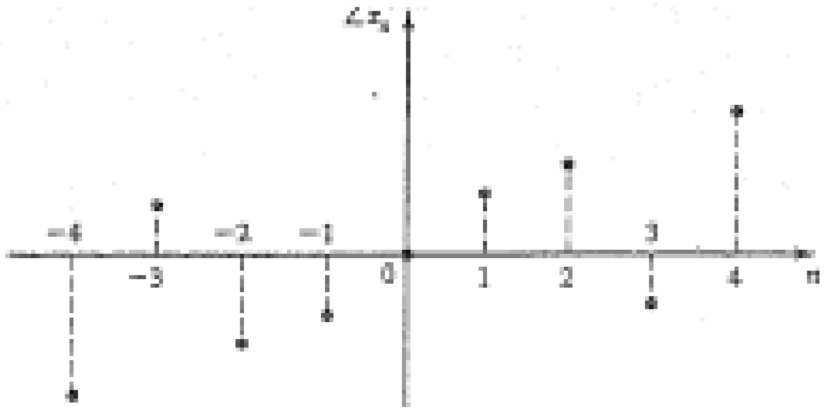
- The spacing between two consecutive spectral coefficients x_n and x_{n+1} is f_0 where f_0 is the fundamental frequency and T_0 is the period.
- It states that the periodic signal $x(t)$ can be described by the period T_0 (or fundamental frequency f_0) and the sequence of complex numbers $\{x_n\}$; i.e., to describe $x(t)$ it is sufficient to specify a *countable* set of (in general complex) numbers. For describing $x(t)$ for all t , we have to specify its values on an *uncountable* set of points.
- In general $x_n = |x_n| e^{j\angle x_n}$. $|x_n|$ gives the magnitude of nth harmonic and $\angle x_n$ gives its phase. The discrete spectrum of the periodic signal $x(t)$ is shown in the figure.

The *discrete spectrum of the* continuous-time periodic signal $x(t)$

- Graph of magnitude $\left|x_n\right|$ of various harmonics



- Graph of phase $\angle x_n$ of various harmonics



Fourier series (cont...)

- The Fourier series expansion can be expressed in terms of angular frequency $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$, by

Synthesis equation: $$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t} \qquad (3)$$

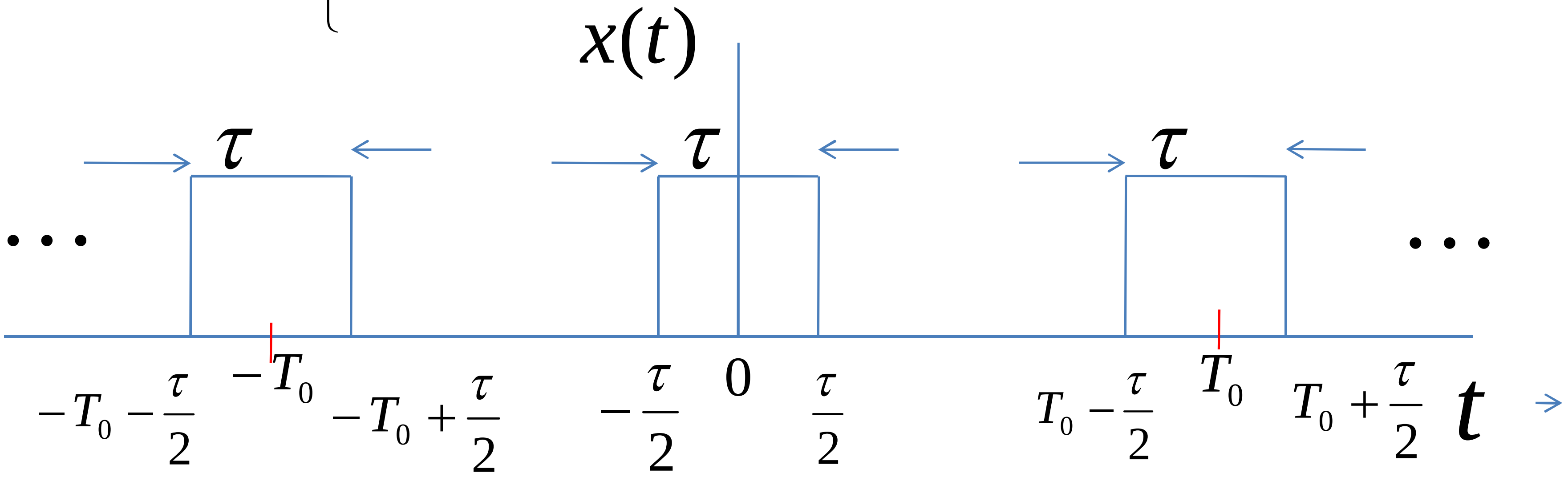
Analysis equation: $$x_n = \frac{\omega_0}{2\pi} \int_{\alpha}^{\alpha + T_0} x(t) e^{-jn\omega_0 t} dt \qquad (4)$$

Periodic signal $x(t)$

A *periodic pulse train* $x(t)$ *described analytically by*

$$x(t) = \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t-nT_0}{\tau}\right) = \sum_{n=-\infty}^{\infty} rect\left(\frac{t-nT_0}{\tau}\right)$$

where $\Pi(t) = \begin{cases} 1 \quad t < \left|\frac{1}{2}\right| \\ \frac{1}{2}, t = \left|\frac{1}{2}\right| \\ 0, otherwise \end{cases}$ *is a rectangular pulse.*



Plot of Fourier Coefficients of periodic Square Wave

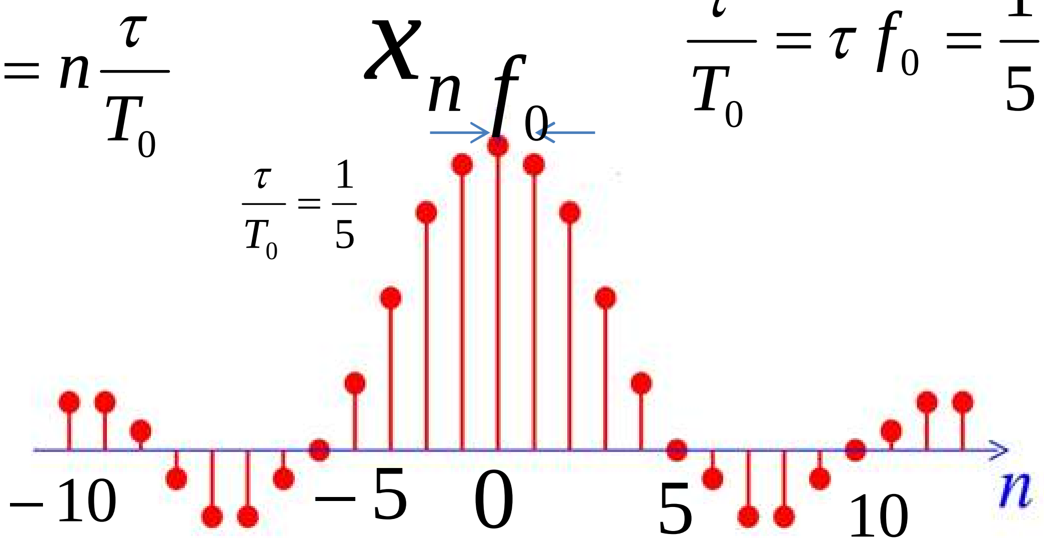
$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$$

$$nf_0 = n\frac{\tau}{T_0}$$

$$\frac{\tau}{T_0} = \tau f_0 = \frac{1}{5}$$

$$x_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$$

$$x_n = \frac{2\sin\left(n\omega_0\tau/2\right)}{n\omega_0 T_0}$$



Or

$$x_n = \frac{\tau}{T_0}\frac{\sin\left(n\pi\frac{\tau}{T_0}\right)}{(n\pi\frac{\tau}{T_0})}$$

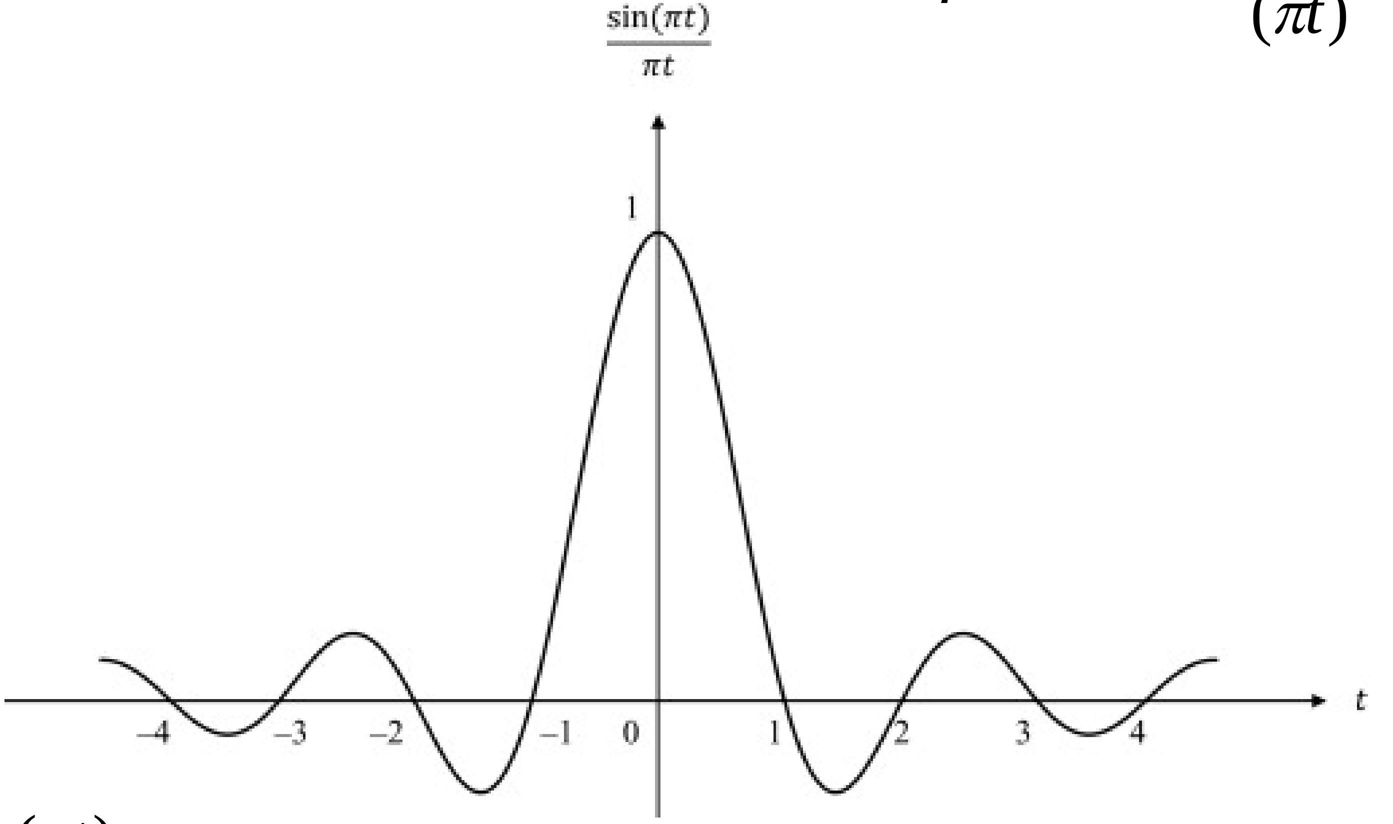
180^0

Or

$$x_n = \frac{\tau}{T_0}\sin c(n\frac{\tau}{T_0}) = \frac{\tau}{T_0}\sin c(n\tau f_0)$$

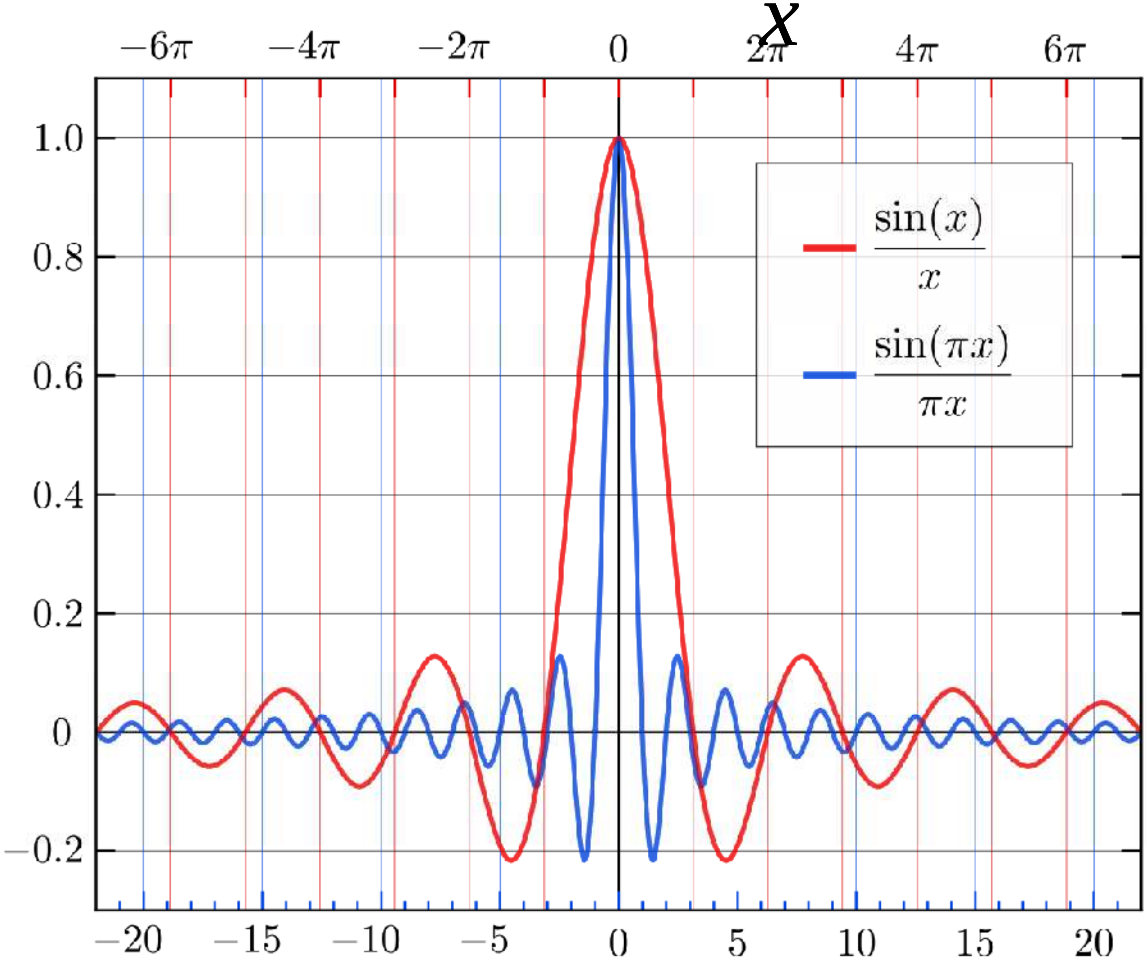
Frequency components are zero at frequencies defined by integer values (other than zero) for $n\tau f_0$ i.e. at harmonic frequency $nf_0 = \pm k\frac{1}{\tau}$, for k is an integer and $k \neq 0.$

Sinc function, $\sin c(t) = \frac{\sin(\pi t)}{(\pi t)}$



$\sin c(t) = \frac{\sin(\pi t)}{(\pi t)}$ becomes zero for $t = \pm n$ where n is an integer other than zero.

Comparison between $\frac{\sin x}{x}$ and $\frac{\sin \pi x}{\pi x}$



$\frac{\sin x}{x}$ becomes zero for $x = \pm n\pi$, where $n = \pm 1, \pm 2 \cdot\cdot$

$\sin cx = \frac{\sin \pi x}{\pi x}$ becomes zero for $x = \pm n \;\; for \;\; n \neq 0$.

Fourier Series for Real Signals: the Trigonometric Fourier Series

- If the signal $x(t)$ *is a real periodic signal satisfying the Dirichlet s conditions, the positive and negative coefficients are complex* conjugates i.e. $x_{-n} = x_n^*$ Hence, $|x_n|$ *has even symmetry and* $\angle x_n$ *has odd symmetry with respect* to the $n = 0$ *axis.*

Let $x_n = \frac{a_n - jb_n}{2}, \quad x_{-n} = \frac{a_n + jb_n}{2}$ *and* $x_0 = \frac{a_0}{2}.$

Then

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos\left(2\pi \frac{n}{T_0} t\right) + b_n \sin\left(2\pi \frac{n}{T_0} t\right) \quad]$$

Three alternative ways to represent the Fourier series expansion of a real periodic signal

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t} \qquad .....(5)$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(2\pi \frac{n}{T_0} t\right) + b_n \sin\left(2\pi \frac{n}{T_0} t\right) \right] \qquad .....(6)$$

$$= x_0 + 2\sum_{n=1}^{\infty} \left| x_n \right| \cos\left(2\pi \frac{n}{T_0} t + \angle x_n\right) \qquad .....(7)$$

Three alternative ways to represent the Fourier series expansion of a real periodic signal(cont...)

where the corresponding coefficients are obtained from

$$x_n = \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)e^{-j2\pi\frac{n}{T_0}t}\,dt = \frac{a_n - jb_n}{2},$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos\left(2\pi\frac{n}{T_0}t\right)dt,$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin\left(2\pi\frac{n}{T_0}t\right)dt,$$

$$\left|x_n\right| = \frac{1}{2}\sqrt{a_n^2 + b_n^2},$$

$$\angle x_n = -\arctan\left(\frac{a_n}{b_n}\right).$$

Summary of Fourier Series representation of CT Periodic signals

- For periodic signals, complex exponential building blocks $e^{j2\pi n f_0 t}$ are harmonically related and linear combination of complex exponentials takes the form of sum
- The spacing between two consecutive spectral coefficients X_n and X_{n+1} is f_0 where f_0 is the fundamental frequency and T_0 is the period
- As the period T_0 increases, fundamental frequency f_0 decreases and harmonically related spectral components become closer in frequency
- $f_0 \to 0$ as $T_0 \to \infty$ and the spectral components form a continuum and the Fourier Series sum takes the form of *integral.*

References

1. John G.Proakis, M. Salehi, COMMUNICATION SYSTEMS ENGINEERING, 2nd ed. New Delhi, India: PHI Learning Private Limited, 2009

Thank you!

Any queries?