LECTURE NOTE

On

Fundamentals of Unit Operation

(3rd semester)

Subject code – HNCE0201

Prepared By

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HNCE0201	Fundamentals of Unit Operations	3L-1T-0P	4 Credits

Objective of the course:

This course will enable students

1. To know the fundamental concepts of fluid mechanics, heat and mass transfer.
2. To solve the engineering problems related to fluid flow, heat and mass transfer.
3. To understand the design concepts of fluid and particulate technology.

Module-1(10 Hours)

Fluid definition and classification, Rheological behavior of fluids & Newton's Law of viscosity. Fluid statics-Pascal's law, Hydrostatic equilibrium, Barometric equation and pressure measurement(problems),Basic equations of fluid flow – Continuity equation, Euler's equation and Bernoulli equation; Types of flow – laminar and turbulent; Reynolds experiment; Flow through circular and non circular conduits – Hagen Poiseuille equation (no derivation).Flow past immersed bodies – drag and drag co-efficients, application of KozneyKarmen & Burke Plummer equation; Flow through stagnant fluids – theory of Settling and Sedimentation – Equipments (cyclones, thickners) Conceptual numericals.

Module-2 (10 Hours)

Different types of flow measuring devices, flow measurements – Orifice meter, Venturimeter, Rotameter. Pumps – types of pumps (Centrifugal & Reciprocating pumps), application of Bernoulli's equation for Energy calculations in pumps.Properties and handling of particulate solids – characterization of solid particles, average particle size, screen analysis- Conceptual numericals of differential and cumulative analysis. Size reduction –characteristics of comminuted products, crushing laws, working principle of ball mill., Mixing – types of mixers (ribbon and muller mixer), power number and power number calculation; Filtration & types, filtration equipments (plate and frame, rotary drum). Conceptual numericals.

Module-3 (10 Hours)

Modes of heat transfer; Conduction – steady state heat conduction through unilayer and multilayer walls, cylinders; Insulation, critical thickness of insulation. Convection- Forced and Natural convection, principles of heat transfer co-efficents, log mean temperature difference, individual and overall heat transfer co-efficients, fouling factor; Condensation – film wise and drop wise (no derivation). Conceptual numericals.

Module-4 (10 Hours)

Heat transfer equipments – double pipe heat exchanger, shell and tube heat exchanger.
Diffusion – Fick's law of diffusion. Types of diffusion. Steady state molecular diffusion in fluids at rest and laminar flow (stagnant / unidirection and bi direction). Mass, heat and momentum transfer analogies. Measurement of diffusivity, Mass transfer coefficients and their correlations. Interphase mass transfer- equilibrium, diffusion between phases. Conceptual numericals.

Text Books

1. Unit operations in Chemical Engineering by Warren L. McCabe , Julian C. Smith & Peter Harriot, McGraw-Hill Education (India) Edition 2014.
2. Transport Process Principles and Unit Operations by Christie Geankoplis, Prentice Hall of India.
3. Fluid Mechanics by K L Kumar, S Chand & Company Ltd.
4. Introduction to Chemical Engineering by Badger W.I. and Banchero, J.T., Tata McGraw Hill New York, 1997.
5. Mass Transfer Operations by Robert E. Treybal. McGraw-Hill Education

Course Outcomes:

Course outcomes:
After studying this course, students will be able to

1. State and describe the nature and properties of the fluids.
2. Study the different flow measuring instruments.
3. Study and understand the principles of various size reduction, conveying equipments, sedimentation and mixing tanks.
4. Comprehend the laws governing the heat and mass transfer operations to solve the problems.

Module-3

Introduction

- *Heat transfer* is the science that seeks to predict the energy transfer that may take place between the material bodies as a result of temperature different.
- So the science of heat transfer seeks not only to explain how energy may be transferred but also to predict the rate at which it exchange will take place under certain specified condition.

Modes of Heat Transfer

There are three modes of heat transfer

- Conduction
- Convection
- Radiation

Conduction

If a temperature gradient exits in a continuous substance, heat can flow unaccompanied by any observable motion of matter. Heat flow of this kind is called *conduction.*

When a temperature gradient exit in a body there is an energy transfer from the high temperature region to the lower temperature region, that energy is transfer by conduction. The heat transfer per unit area is proportional to the temperature gradient.

Mathematically it's denoted as,

$$\frac{Q}{A} \propto -\frac{\partial T}{\partial x} \Rightarrow Q = -KA\frac{\partial T}{\partial x}$$

This is known as **Fourier's Law of Heat Conduction**.

Here Q= q/A

Where q= Heat flux, (W/m^2)

A= Area through which heat transfer occurs, (m^2)

Basically it is the heat transfer rate in the x direction per unit area perpendicular to the direction of transfer, and it is proportional to the temperature gradient, $\frac{\partial T}{\partial x}$ in this direction. The parameter ***K*** is a transport property known as the thermal conductivity (W/m. K) and is a characteristic of the wall material. The ***minus (-) sign*** is a consequence of the fact that hat is transferred in the direction of decreasing temperature.

Steady state one dimensional heat conduction

Consider a flat slab of thermal conductivity k and heat transfer area Ax is the length measure from the hot side of the wall

Assumption

- K is independent of temperature
- The area of the wall is very large in comparison to its thickness so that heat losses from the edges are negligible.
- The direction of heat flow is perpendicular to the wall

Q/A = -k$\frac{\partial T}{\partial x}$

⇨ $\int_{T_1}^{T_2} dT = -\frac{Q}{kA}\int_{x_1}^{x_2} dx$

⇨ $\frac{Q}{A} = k\frac{\Delta T}{B}$

⇨ $\frac{Q}{A} = \frac{\Delta T}{R}$

Where **R** = resistance across the slab

ΔT = difference in temperature

Compound Resistance in series

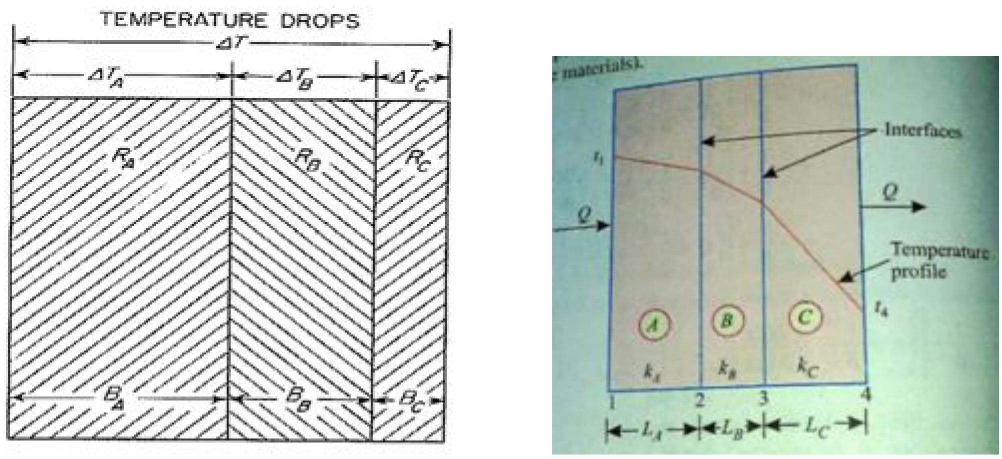


Figure 1 Compound wall consists of three layers

Consider a flat slab consisted of a series of layers.

Let be B_A ,B_B,B_C are the thickness of layers A, B, C.

ΔT_A, ΔT_B, ΔT_C are the temperature drop across the layers A,B ,C.

A = area of the compound wall which are perpendicular to the plane of heat transfer

$\Delta T = total\ temperature\ drop\ across\ the\ entire\ wall$

$$\Delta T = \Delta T_A + \Delta T_B + \Delta T_C$$

$\Delta T_A = \frac{Q_A B_A}{K_A A}$, $\Delta T_B = \frac{Q_B B_B}{K_B A}$, $\Delta T_C = \frac{Q_C B_C}{K_C A}$

$\Delta T = \frac{1}{A}\left[\frac{Q_A B_A}{K_A A} + \frac{Q_B B_B}{K_B A} + \frac{Q_C B_C}{K_C A}\right]$

$\Delta T = \frac{Q}{A}\left[\frac{B_A}{K_A} + \frac{B_B}{K_B} + \frac{B_C}{K_C}\right]$

R = R_A+ R_B + R_C

So

$\boldsymbol{\frac{\Delta T_A}{R_A} = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{R_C}}$

Steady state heat conduction through a cylinder of variable area

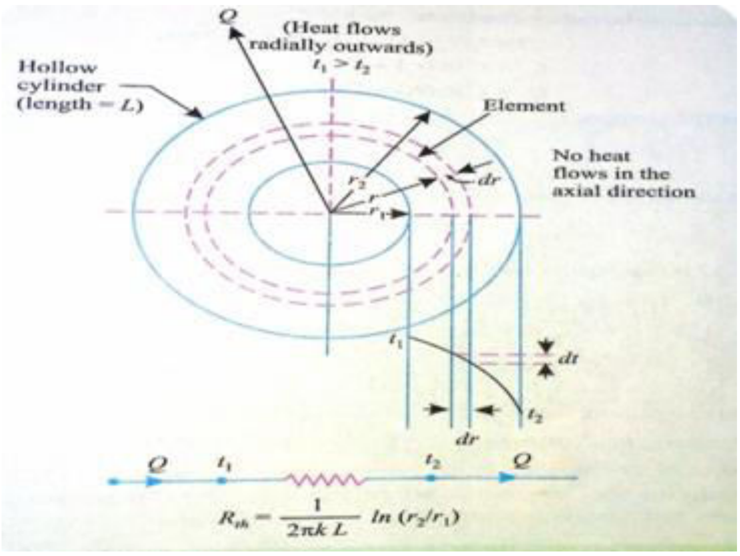


Figure 2 A cylinder of variable area

Let's consider a hollow cylinder of inside radius 'r_i' and outer radius 'r_o' and length 'L'

Assumption

The heat flow occurs is the radial direction

Heat flux (q) = $\frac{Q}{A}$ so Q = qA

At steady state condition, there can't be any accumulation of heat, if we make a heat balance over a thin cylindrical cell of radius r and thickness Δr .

So rate of heat input must be equal to the rate of heat output.

$$q_r A = q_{r+\Delta r}$$

$$\Rightarrow 2\pi rl \frac{q_r}{r} = 2\pi rl \frac{q_r}{r + \Delta r}$$

$$\Rightarrow 2\pi rl \frac{q_r}{r} - 2\pi rl \frac{q_r}{r+\Delta r} = 0$$

$$\Rightarrow \lim_{\Delta r \to 0} \frac{2\pi rl \frac{q_r}{r+\Delta r} - 2\pi rl \frac{q_r}{r}}{\Delta r} = 0$$

$$\Rightarrow \frac{d}{dr} (rq_r) = 0$$

$$\Rightarrow r = (-k \frac{dT}{dr}) = c_1$$

Integrating it form starting point to end point we get,

$$\int_0^T dT = -\frac{C_1}{k} \int_0^r \frac{dr}{r}$$

$$\Rightarrow T = -\frac{C_1}{k} \log r$$

Putting boundary condition

1. ***T= T_i , r= r_i***
2. ***T= T_0 , r= r_0***

Now we get,

$$T_i - T_0 = -\frac{C_1}{k} \ln r_i + \frac{C_1}{k} \ln r_0 \Rightarrow c_1 = K \frac{(T_i - T_0)}{\ln \frac{r_i}{r_0}}$$

Now putting value of C_1 we get, $C_2 = T_i + \frac{(T_i - T_0)}{ln\frac{r_i}{r_0}} \; lnr_i$

As we know

$rq_r = c_1$ And Q = qA

So

$$\mathbf{Q = 2\pi lk \frac{(T_i - T_0)}{ln\frac{r_i}{r_0}}}$$

Steady state heat conduction through a Sphere of variable area

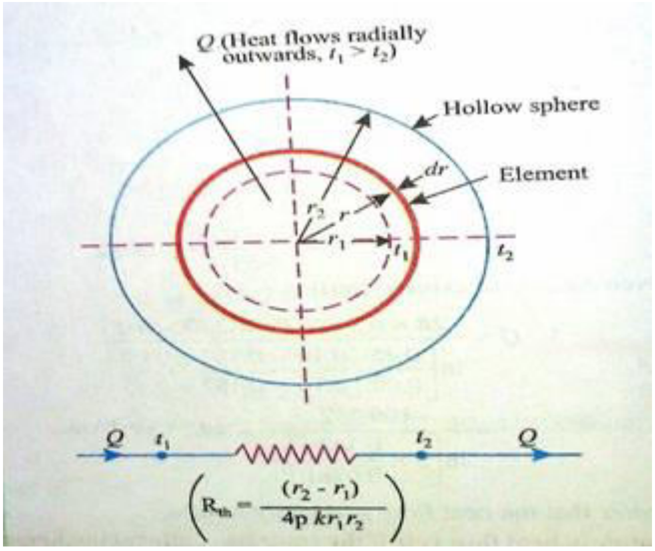


Figure 3 A sphere of variable area

Let's consider a hollow sphere of inside radius 'r_i' and outside radius 'r_0'.

The heat flow occurs in radial direction and

Heat flux (q) $= \frac{Q}{A} \Rightarrow Q = qA$

We know rate of heat input must be equal to the heat output

$$q_r A = q_{r+\Delta r}$$

$$\Rightarrow 4\pi r^2 \frac{q_r}{r} = 4\pi r^2 \frac{q_r}{r+\Delta r}$$

$$\Rightarrow 4\pi r^2 \frac{q_r}{r} - 4\pi r^2 \frac{q_r}{r+\Delta r} = 0$$

$$\Rightarrow \lim_{\Delta r \to 0} \frac{4\pi r^2 \frac{q_r}{r+\Delta r} - 4\pi r^2 \frac{q_r}{r}}{\Delta r} = 0$$

$$\Rightarrow \frac{d}{dr}\,(4\pi r^2\, q_r) = 0$$

$$\Rightarrow r^2\ (-k\frac{dT}{dr}) = c_1$$

Integrating it form starting point to end point we get,

$$\int_0^T dT = -\frac{C_1}{k}\int_0^r \frac{dr}{r^2}$$

$$\Rightarrow T = -\frac{C_1}{k}\{-\frac{1}{r^2}\} + c_2$$

Putting boundary condition

1. T= T_i , r= r_i
2. T= T_0 , r= r_0

Now we get, $T_i - T_0 = \frac{C_1}{k}\ (\frac{1}{r_i} - \frac{1}{r_0}) \Rightarrow c_1 = K\frac{(T_i - T_0)}{(\frac{1}{r_i} - \frac{1}{r_0})}$

As you know,

$r^2\, q_r = C_1$

Q = $q_r A$

And A = $4\pi r^2$

So

$$\mathbf{Q} = \frac{\boldsymbol{4\pi k\ (T_i - T_0) r_i r_0}}{\boldsymbol{r_i - r_0}}$$

1-D unsteady state heat conduction of flat slab

Let's consider a small amount of slab of the thickness dx at a distance 'x' from left side of slab. Temperature gradient at 'x' at definite instant of time is $\frac{\partial T}{\partial x}$. The heat input in time interval dt at x is

$$\frac{dQ}{x} = -kA\,\frac{\partial T}{\partial x}\,dt$$

Heat output in the time interval is

$\frac{dQ}{x+dx} = -kA\,[\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)dx]\,dt$ ------1

Accumulation = ρ A dx$\boldsymbol{c_p}\frac{\partial T}{\partial t}\,dt$ ----------2

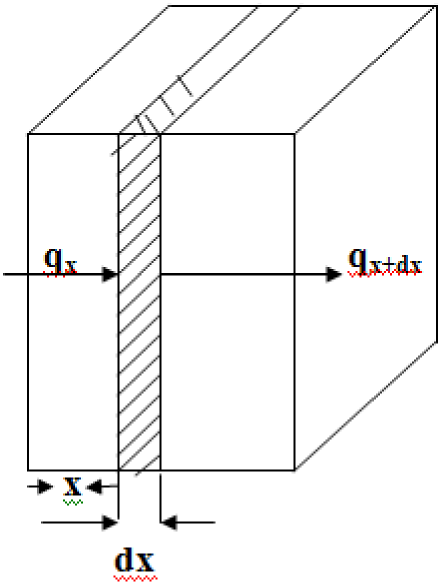
Equating equation 1 & 2 we get,

$$\frac{\partial T}{\partial t} = \alpha\,\frac{\partial^2 T}{\partial x^2}$$

For 3-D heat equation,

$\frac{\partial T}{\partial t} = \alpha[\,\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\,]$

Heat transfer by conduction in a three-dimensional body, where the temperature may be changing with time and heat sources are present within the body:



Consider the general case where the temperature may be changing with time and heat sourses may be present within the body.

By making an energy balance around the element of thickness dx:

Energy conducted in left face + heat generated within the element = Change in internal energy + energy conducted out from the right face (1)

Energy in the left face , $q_x = -KA\dfrac{\partial T}{\partial x}$

Energy generated within the element = $\dot{q}\,Adx$

Change in internal energy = $\rho cA\dfrac{\partial T}{\partial \tau}dx$

Energy out from the right face, $q_{x+dx} = -KA\left.\dfrac{\partial T}{\partial x}\right|_{x+dx} = -A\left[K\dfrac{\partial T}{\partial x} + \dfrac{\partial}{\partial x}\left(K\dfrac{\partial T}{\partial x}\right)dx\right]$

Where, $\dot{q}$ = energy generated per unit volume.

C = specific heat of the material.

ρ = density of the material.

Putting all the above values in equation no-1 and solving, we get:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\dot{q}=\rho c\frac{\partial T}{\partial \tau} \qquad (2)$$

Which is the unsteady state heat conduction equation in one dimension.

If we will consider the heat conduction equation in all the three direction, then the equation becomes:

$$q_x+q_y+q_z+q_{gen}=q_{x+dx}+q_{y+dy}+q_{z+dz}+\frac{\partial E}{\partial \tau} \qquad (3)$$

Where, $q_x=-Kdydz\frac{\partial T}{\partial x}$

$$q_{x+dx}=-\left[K\frac{\partial T}{\partial x}+\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dx\right]dydz$$

$$q_y=-Kdxdz\frac{\partial T}{\partial y}$$

$$q_{y+dy}=-\left[K\frac{\partial T}{\partial y}+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)dy\right]dxdz$$

$$q_z=-Kdxdy\frac{\partial T}{\partial z}$$

$$q_{z+dz}=-\left[K\frac{\partial T}{\partial z}+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)dz\right]dxdy$$

$$q_{gen}=\dot{q}\,dxdydz$$

$$\frac{dE}{d\tau}=\rho cdxdydz\frac{\partial T}{\partial \tau}$$

So that the general three-dimensional heat conduction equation becomes:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\frac{\partial T}{\partial \tau}$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

General heat conduction equation in cylindrical co-ordinate

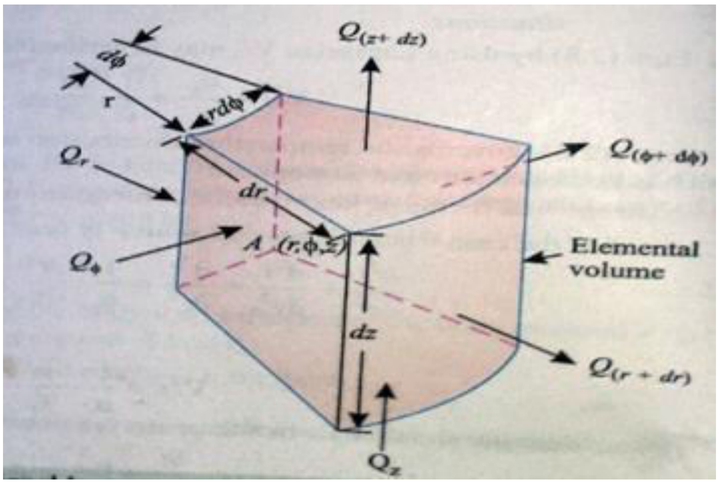


Figure 4 Heat conduction analyses in a cylindrical co-ordinates

The volume of the element = $\mathbf{r\ d\emptyset.dr.dz}$

q_g = heat generation per unit volume per unit time

K = thermal conductivity, **ρ** = density, c_p = specific gravity

In (x -∅) plane i.e. in radial direction,

$$Q_r' = -k\,(r\,d\emptyset.dz)\frac{\partial T}{\partial r}\,d\tau$$

$$Q_{(r+dr)}' = Q_r' + \frac{\partial}{\partial r}(Q_r')\,dr$$

Subtracting equation (ii) from (i) we get

$$dQ_r' = Q'_{(r+dr)} - Q_r'$$

$$= k\,(d\emptyset.dr.dz)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)d\tau$$

$$= k\,r\,(d\emptyset.dr.dz)\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right)d\tau$$

Similarly,

In r-z plane i.e. in tangential direction,

$$dQ_\emptyset' = k\,r\,(d\emptyset.dr.dz)\frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2}\,d\tau$$

In r - ∅plane i.e. in axial direction,

$\mathbf{d}\boldsymbol{Q_z}'$= **k r** (d∅. $\boldsymbol{dr.dz}$) $\frac{\partial^2 t}{\partial z^2}$ **d**$\boldsymbol{\tau}$

Net heat accumulation = **k r (d∅.** $\boldsymbol{dr.dz}$) [$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2} + \frac{\partial^2 t}{\partial z^2}$] **d**$\boldsymbol{\tau}$

Total heat generation ($\boldsymbol{Q_g}'$) = $\boldsymbol{q_g}$ **(r dr dz d∅) d**$\boldsymbol{\tau}$

Energy stored = **ρ (r d∅.** $\boldsymbol{dr.dz}$) $\boldsymbol{c_p}\frac{\partial t}{\partial \tau}$ **d**$\boldsymbol{\tau}$

So as we know

Net heat accumulation + total heat generated = Energy stored

So mathematically,

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho c}{k}\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

General heat conduction equation in Spherical co-ordinate

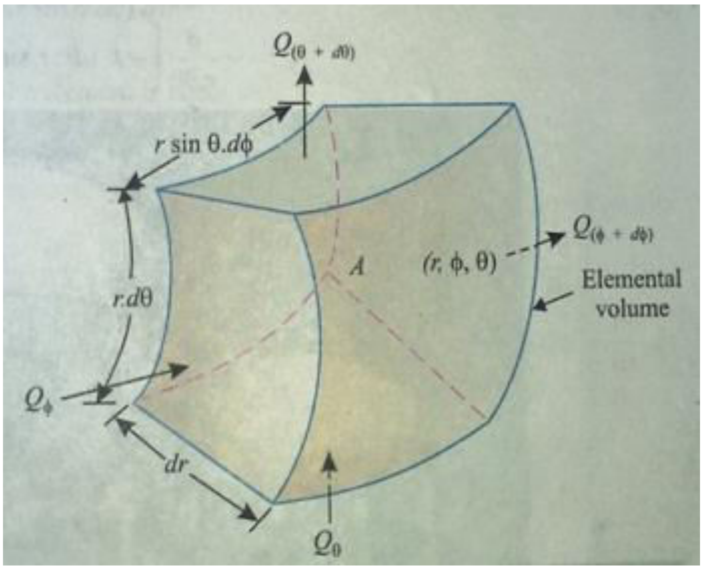


Figure 5 Heat conduction analyses in spherical co-ordinates

The volume of the element = **r sin** θ **d∅.** $\boldsymbol{dr.rd\theta}$

$\boldsymbol{q_g}$ = Heat generation per unit volume per unit time

K = thermal conductivity, **ρ** = density, $\boldsymbol{c_p}$ = specific gravity

In (r -$\boldsymbol{\theta}$) plane i.e. in tangential direction,

$Q_{\emptyset}'$= - k (r d$\theta . dr$) $\frac{\partial t}{r \sin\theta\, \partial\emptyset}$ dT ----------i

$Q_{(\emptyset + d\emptyset)}{}' = Q_{\emptyset}' + \frac{\partial}{r \sin\theta\, \partial\emptyset}(Q_{\emptyset}')$ r sin θ. d∅------ii

Subtracting equation (ii) from (i) we get

d$Q_{\emptyset}' = Q'_{(\emptyset+d\emptyset)} - Q_{\emptyset}$'

$= \mathbf{k}\ (\mathbf{r} \sin\theta\ \mathbf{d}\emptyset. dr. rd\theta) \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \emptyset^2}\, \mathbf{d}\tau$

Similarly,

In r -∅ plane i.e. in radial direction,

$\mathbf{d}Q_{\theta}' = \mathbf{k}\ (\mathbf{r} \sin\theta\ \mathbf{d}\emptyset. dr. rd\theta) \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} [\sin\theta\ \frac{\partial t}{\partial\theta}]\ \mathbf{d}\tau$

In ∅ − θplane i.e. in axial direction,

$\mathbf{d}Q_{r}' = \mathbf{k}\ (\mathbf{r} \sin\theta\ \mathbf{d}\emptyset. dr. rd\theta) \frac{1}{r^2} \frac{\partial}{\partial r} [\, r^2\ \frac{\partial t}{\partial r}]\ \mathbf{d}\tau$

Net heat accumulation = $\mathbf{k}\ (\mathbf{r} \sin\theta\ \mathbf{d}\emptyset. dr. rd\theta)\ [\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \emptyset^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta\ \frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2} \frac{\partial}{\partial r}(\, r^2\ \frac{\partial t}{\partial r})]\ \mathbf{d}\tau$

Total heat generation (Q_g') = $q_g\ (\mathbf{r} \sin\theta\ \mathbf{d}\emptyset. dr. rd\theta)\ \mathbf{d}\tau$

Energy stored = $\rho\ (\mathbf{r} \sin\theta\ \mathbf{d}\emptyset. dr. rd\theta)\ c_p \frac{\partial t}{\partial\tau}\ \mathbf{d}\tau$

So as we know

Net heat accumulation + total heat generated = Energy stored

So mathematically,

$$\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \emptyset^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta\ \frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2} \frac{\partial}{\partial r}(\, r^2\ \frac{\partial t}{\partial r}) + \frac{q_g}{k} = \frac{\rho c}{k} \frac{\partial t}{\partial\tau} = \frac{1}{\alpha} \frac{\partial t}{\partial\tau}$$

Critical thickness of insulation

We know that adding more insulation to a wall or to the attic always decreases heat transfer. The thicker is the insulation; the lower is the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance. Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

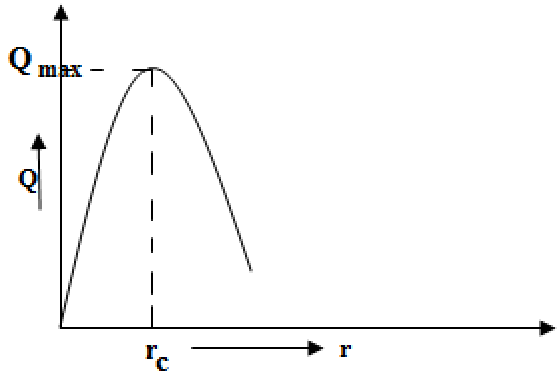


Figure 6 Critical radius of insulation

Critical radius of cylinder

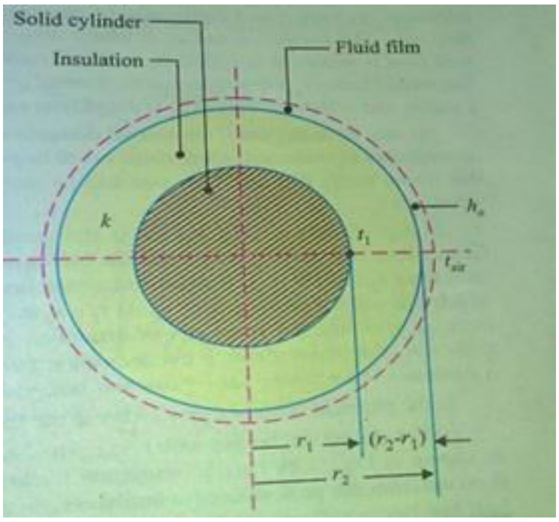


Figure 7 Critical insulation thicknesses for a cylinder

Let's consider a solid cylinder of radius $\boldsymbol{r_1}$insulated with an insulation of thickness ($\boldsymbol{r_2 - r_1}$) in the above figure.

L = length of the cylinder
$\boldsymbol{t_1}$= surface temperature of the cylinder
$\boldsymbol{t_{air}}$ = temperature of air
$\boldsymbol{h_0}$= heat transfer co-efficient at the outer surface of the insulation
K = thermal conductivity of insulating material

Then the rate of heat transfer from the surface of the solid cylinder to the surrounding is given by

$$\mathbf{Q} = 2\pi l \frac{(t_1 - t_{air})}{\frac{\ln\frac{r_2}{r_1}}{k} + \frac{1}{h_0 2\pi r_2 l}}$$

From the above equation we get,

If $r_2$increases then factor $\frac{ln\frac{r_2}{r_1}}{k}$increases but $\frac{1}{h_0 r_2}$decreases

So Q becomes maximum when,

$$\frac{\partial}{\partial r_2}[\frac{ln\frac{r_2}{r_1}}{k} + \frac{1}{h_0 2\pi r_2 l}] = 0$$

Solving this finally we get

K = $h_0 r_2$

$$\boxed{r_2 = \frac{K}{h_0}}$$

Critical radius of sphere

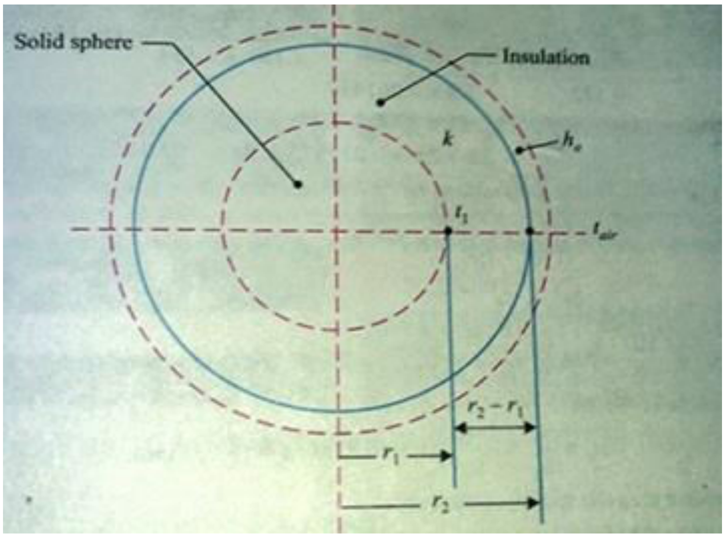


Figure-8 Critical insulation thickness for a sphere

Let's consider a sphere of radius $r_1$insulated with an insulation of thickness ($r_2 - r_1$) in the above figure.

A = Surface area of the sphere
t_1= surface temperature of the sphere
t_{air} = temperature of air

$\boldsymbol{h_0}$= heat transfer co-efficient at the outer surface of the insulation
K = thermal conductivity of insulating material

Then the rate of heat transfer from the surface of the sphere to the surrounding is given by

$$Q = \frac{t_1 - t_{air}}{\frac{r_2 - r_1}{4\pi K r_1 r_2} + \frac{1}{4\pi r_2^2 h_0}}$$

So Q becomes maximum when,

$$\frac{d}{dr_2}\left[\frac{r_2 - r_1}{4\pi K r_1 r_2} + \frac{1}{4\pi r_2^2 h_o}\right] = 0$$

$$\Rightarrow \frac{d}{dr_2}\left[\frac{r_2 - r_1}{K r_1 r_2} + \frac{1}{r_2^2 h_0}\right] = 0$$

$$\Rightarrow \frac{d}{dr_2}\left[\frac{1}{K r_1} - \frac{1}{K r_2} + \frac{1}{r_2^2 h_0}\right] = 0$$

$$\Rightarrow \frac{1}{K r_2^2} - \frac{2}{r_2^3 h_0} = 0$$

$$\Rightarrow 2K r_2^2 = r_2^3 h_0$$

$$\Rightarrow \boxed{r_2 = r_c = \frac{2k}{h_0}}$$

Convection

Whenever a solid body exposed to a moving fluid having a temperature different from that of the body, energy is carried or convected from or to the body by the fluid.

Convection is the heat transfer between solid surface and fluid system in motion is

$$Q = hA\Delta T$$

Where, Q=rate of heat flow across surface
h=individual heat transfer coefficient
A=area of heat transfer
ΔT=Temperature difference between two surface

Convection is divided into types:-

1. Natural convection
2. Forced convection

1. Natural convection

Natural convection is the mechanism or type of heat transfer, in which the fluid motion is generated by density differences in the fluid occurring due to temperature gradients.

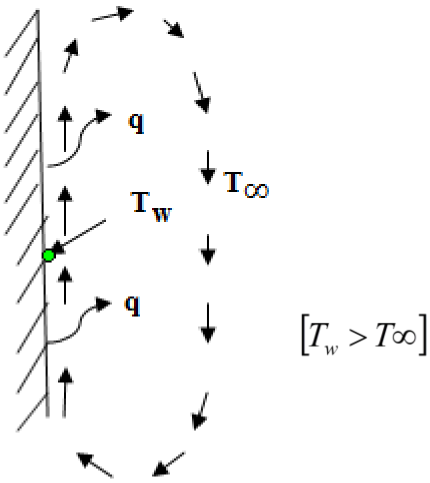


Figure 1 Natural or free convection of air

Let the plate is maintained at the isothermal temperature T_w which is higher than the surrounding fluid temperature T_∞. So the fluid near the wall on getting heated, moves up due to the effect of buoyancy and is raplaced by the cold fluid moving towards the wall. Thus a circulation current is set up due to the density difference.

In natural convection Nusselt number is a function Prandtl number and grassoff's number i.e.

$$Nu = f(Pr, Gr)$$

2. Force convection

Force convection is the mechanism or type of heat transfer, in which fluid motion is generated by external source (like a pump, fan, suction device etc).

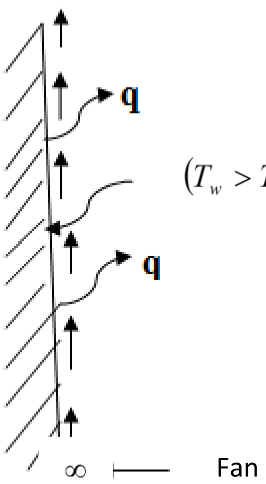


Figure 2 Forced convection of air

When the mass motion of the fluid is caused by an external device, like a pump, compressor, blower or fan, the process is called forced convection. Here fluid is made to flow along the hot surface due to the pressure difference generated by the device and heat is transferred from the wall to the fluid.

In forced convection Nusselt number is a function Prandtl number and Reynold's number i.e.

$$Nu=f(Re,Pr)$$

Newton′s laws of cooling

According to this law, the rate of heat flux is directly propotional to temperature difference .

Regimes of heat transfer in fluids

A fluid is being heated or cooled may be flowing in laminar flow ,in turbulent flow or in transition range between laminar and turbulent flow. Also fluid will flow in forced & natural convection. The direction of flow of fluid may be parallel to the heating surface, so that boundary layer thickness is least prominent or if the direction of flow is perpendicular or at an angle to the heating surface. Then the boundary layer separation takes place ,because of condition of flow at entrance of a tube differ from those well downstream from the entrance, the velocity field and the assosciated temperature field may depend from the distance of tube entrance and also depends upon situation , fluid flow through a preliminary length of unheated or un cooled pipe, so that fully developed velocity field is established .The temperature field is created within an existing velocity field.

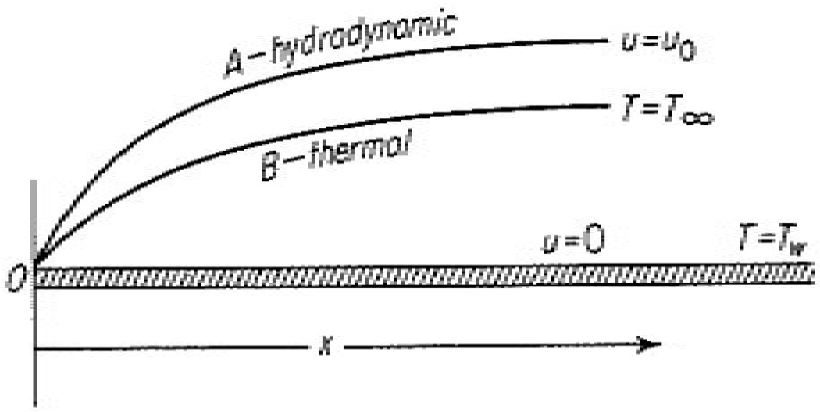


Figure 3 thermal and hydrodynamic boundary layers on a entirely heated flat plate

Hydrodynamic boundary layer

A boundary layer developes within which the velocity varies from u=0 to u=u_0 at the outer boundary layer.This boundary layer is called hydrodynamic boundary layer and shown by above figure.

Thermal boundary layer

The penetration of heat by the transfer from the plate to the fluids change the temperature of the fluid near the surface of the plate and temperature gradient is generated.The temperature gradient is confined to a layer next to the wall and within the layer,temperature varies from T=T_w to T=T_∞.The boundary layer is called thermal boundary layer as shown by above figure.

The thermal boundary layer thinner than hydrodynamic boundary layer at all distance from leading edge of X.

Relationship between thickness of two boundary layer at a given point along the plate depends on the dimensionless number is known as prandtl number (Pr).

Mathematically,

$$Pr = \frac{hydrodynamic\ boundary\ layer}{thermal\ boundary\ layer}$$

For, Pr>1, TBL<HBL (eg.-most liquids)

Pr=1, TBL=HBL (eg.-gases)

Pr<1, TBL>HBL (eg.-liquid metals)

Individual overall heat transfer coefficient

In the above figure the metal wall of the tube separate the warm fluid on the right to the cold fluid on the left. The change in temperature with the distances shown by the line $T_a, T_b, T_{wh}, T_{wc}, T_e, T_g$.Thus the temperature profile is divided into three parts:-

- Through the metal wall and,
- Through each of the two fluid

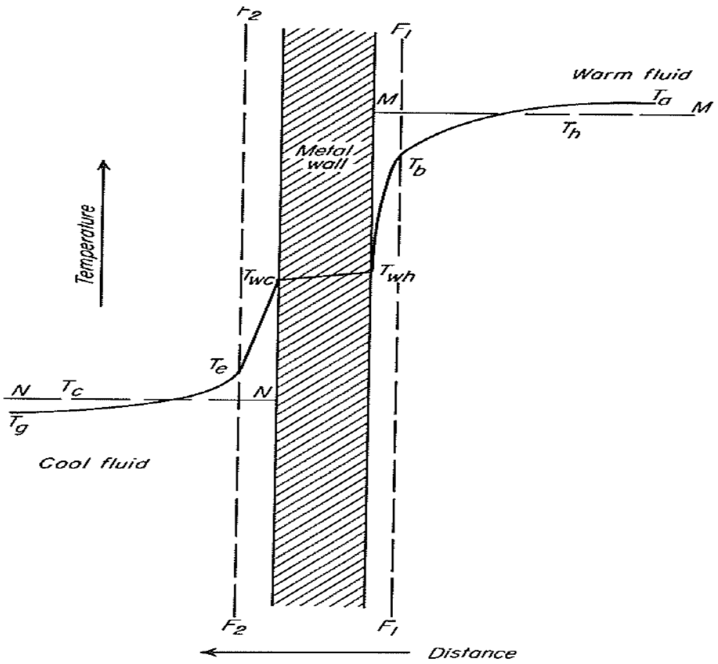


Figure 5 Temperature gradient in forced convection

The overall effect is therefore study in terms of individual parts. When one fluid flows through a single pipe three zones are visible:-

- Laminar sub layer
- Buffer zone
- Turbulent core

The velocity gradient is large near the wall, small in the turbulent core and rapid change in the buffer layer. Similarly in the case of the temperature gradient it is larger at the wall (through laminar sub layer), small in the turbulent core and rapid change in the buffer zone.

The overall resistance to the flow of heat from hot fluid to cold fluid is a result of three separate resistances operating in series.

The individual or surface heat transfer coefficient h defined by the equation:-

$$h = \frac{\frac{dQ}{dA}}{T - T_w} \quad (1)$$

$\frac{dQ}{dA}$=local heat flux based on the area in contact with fluid.
T=local average temperature of the fluid
T_w=Temperature of the wall in contact with the fluid.

Heat transfer by conduction is given by:-

$$\frac{dQ}{dA} = -K(\frac{dT}{dY})_w \quad (2)$$

Normal distance measured into fluid from wall

$$h_i = \frac{(\frac{dT}{dY})_w}{T-T_w} \quad (3)$$

$$\text{Nu}=\frac{-D(\frac{dT}{dY})_w}{T-T_w} \quad (4)$$

Dimensionless group that is nusset number is the ratio of temperature gradient near the wall to the temperature gradient across entire pipe.
Another interpretation of nusset number can be obtained by considering the gradient that would exist if all the resistance to heat transfer were in a laminar layer in which heat transfer was only by conduction.

$$\frac{dQ}{dA}=\frac{-K(T-T_w)}{x}= \text{h } (T\text{-}T_w) \quad (5)$$

$$\text{h}=\frac{k}{x}$$

$$\text{Nu}=\frac{hD}{x} = \frac{D}{x} \quad (6)$$

Therefore,this nusselt number is the ratio of the tube diameter to the equivalent thickness of laminar layer.Sometimes
x=film thickness

$$h_i = \frac{\frac{dQ}{dA_i}}{T_h-T_{wh}} \quad (7)$$

$$h_o = \frac{\frac{dQ}{dA_o}}{T_{wc}-T_c} \quad (8)$$

Where A_i *and* A_o are inside and outside area of the tube.

Calculation of overall heat transfer coefficient from individual heat transfer coefficient:

The rate of heat transfer through the tube wall is given by the differential form,

$$\frac{dq}{d\overline{\overline{A}}_L} = \frac{k_m(T_{wh} - T_{wc})}{x_w} \tag{1}$$

Where, $(T_{wh} - T_{wc}) \rightarrow$ Temperature difference through the tube wall

$k_m \rightarrow$ Thermal conductivity of the wall

$x_w \rightarrow$ Tube wall thickness

$\frac{dq}{d\overline{\overline{A}}_L} \rightarrow$ Local heat flux based on logarithmic mean of inside and outside area of tube.

$$h_i = \frac{\frac{dq}{dA_i}}{T_h - T_{wh}} \tag{2}$$

$$\Rightarrow (T_h - T_{wh}) = dq\left[\frac{1}{h_i dA_i}\right]$$

$$h_o = \frac{\frac{dq}{dA_o}}{T_{wc} - T_c} \tag{3}$$

$$\Rightarrow (T_{wc} - T_c) = dq\left[\frac{1}{h_o dA_o}\right]$$

From eq-1,

$$(T_{wh} - T_{wc}) = dq\left[\frac{x_w}{k_m d\overline{\overline{A}}_L}\right]$$

The temperature difference $(T_h - T_c)$,

$$(T_h - T_c) = (T_h - T_{wh}) + (T_{wh} - T_{wc}) + (T_{wc} - T_c)$$

$$\Rightarrow (T_h - T_c) = dq\left[\frac{1}{h_i dA_i} + \frac{x_w}{k_m d\overline{\overline{A}}_L} + \frac{1}{h_o dA_o}\right] \tag{4}$$

If the heat transfer rate is calculated based on the outside area,

$$\frac{dq}{dA_o} = \frac{(T_h - T_c)}{\frac{1}{h_i}\left(\frac{dA_o}{dA_i}\right) + \frac{x_w}{k_m}\left(\frac{dA_o}{d\overline{\overline{A}}_L}\right) + \frac{1}{h_o}}$$

Again $\frac{dA_o}{dA_i} = \frac{D_o}{D_i}$ and $\frac{dA_o}{d\overline{\overline{A}}_L} = \frac{D_o}{\overline{\overline{D}}_L}$

The eq-6 can be reduced to:

$$\frac{dq}{dA_o} = \frac{(T_h - T_c)}{\frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{x_w}{k}\left(\frac{D_o}{\overline{D_L}}\right) + \frac{1}{h_o}} \tag{5}$$

Also $\frac{dq}{dA} = U\Delta T = U(T_h - T_c)$ (6)

Comparing eq-5 and eq-6,

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{x_w}{k_m}\left(\frac{D_o}{\overline{D_L}}\right) + \frac{1}{h_o}} \tag{7}$$

The overall heat transfer co-efficient based on the outside area of the tube.

Similarly the overall heat transfer co-efficient based on the inside area of the tube is given by:

$$U_i = \frac{1}{\frac{1}{h_o}\left(\frac{D_i}{D_o}\right) + \frac{x_w}{k_m}\left(\frac{D_i}{\overline{D_L}}\right) + \frac{1}{h_i}} \tag{8}$$

Fouling factor

In actual service, the heat transfer surface do not remain clean. Scale, dirt and other solid deposits form on one or both the sides of the tubes ,provide additional resistance to heat flow and reduce the overall coefficient. The effect of such deposits is taken into account by adding a term h_d to equation of overall coefficient.

$$U_o = \frac{1}{\frac{1}{h_o} + \frac{1}{hd_o} + \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{1}{hd_i}\left(\frac{D_o}{D_i}\right) + \frac{x_w}{k_m}\left(\frac{D_o}{\overline{D_L}}\right)} \tag{9}$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{1}{hd_i} + \frac{1}{h_o}\left(\frac{D_i}{D_o}\right) + \frac{1}{hd_o}\left(\frac{D_i}{D_o}\right) + \frac{x_w}{k_m}\left(\frac{D_i}{\overline{D_L}}\right)} \tag{10}$$

Where hd_i and hd_o are the fouling factors for the scale deposits on the inside and outside of tube surfaces respectively.

Condensation

Heat transfer to a surface occurs by condensation when the surface temperature is less than the saturation temperature of an adjoining vapor. A vapor may condense on a cold surface in one of two ways, which are well described by the terms *drop wise* and *film type* condensation.

Film type condensation

- Entire surface is covered by the condensate, which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface.
- Condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity.
- Thickness of the liquid film increases in the flow direction as more vapor Condenses on the film.
- Thermal resistance is reduced through use of short vertical surfaces and horizontal cylinders.
- Characteristic of clean, uncontaminated surfaces.

Drop wise condensation

- Surface is covered by drops ranging from a few micrometers to agglomerations visible to the naked eye.
- Condensed vapor forms droplet on the surface instead of a continuous film, and the Surface is covered by countless droplets of varying diameter.
- Thermal resistance is greatly reduced due to absence of a continuous film.
- Surface coatings may be applied to inhibit *wetting* and stimulate drop wise condensation.

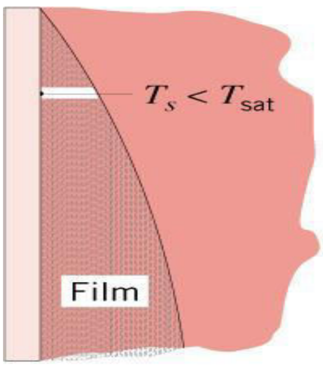


Figure 1 (a)Film type condensation

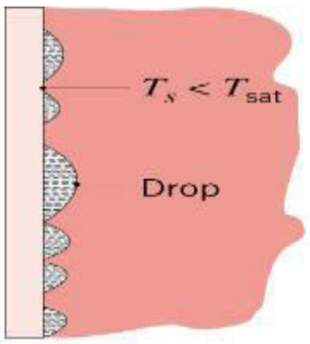


Figure 1 (b) Drop wise condensation

Rate of heat transfer in drop wise condensation is 18 times more than that of film wise condensation. Due to the present of film in film wise condensation it decreases the rate of heat transfer.

Coefficients for film-type condensation

The basic equations for the rate of heat transfer in film-type condensation were first derived by Nusselt. The Nusselt equations are based no the following assumptions.

- The vapor and the liquid at theoutside boundary of the liquid layer are in thermodynamic equilibrium, so that the only resistance to the flow of heat is that offered by the layer of condensate flowing downword in laminar flow under the action of gravity.
- The velocity of the liquid at the wall is zero, that the velocity of the liquid at the outside of the film is not influenced by the velocity of the vapor, and that the temperatures of the wall and the vapor are constant.
- The physical properties of the liquid are taken at the mean film temperature.

Solved Examples

A. Short answer types

1. Walls of acubical oven are are of thickness L, and they are made up of thermal conductivity K. the temperature inside the oven is 100 ^{o}C and the inside heat ransfer coefficient is 3K/L. if the wall temperature on the outside is held at 25 oC, what is the inside wall temperature in degrees C?

   Answer:

   We know $q = \frac{75}{L/3K + L/K} \frac{100 - T_w}{(L/3K)}$

   $$\Rightarrow \frac{75}{4L/3K} = \frac{100 - T_w}{L/3K}$$

   $$\Rightarrow 75/4 = 100 - T_w$$

   $$\Rightarrow T_w = 81.25°C$$

2. The heat flux (from outside to inside) across an insulating wall with thermal conductivity K= 0.04 W/mK and thickness 0.16 m is 10 W/m^2. The temperature of the inside wall is -5^{o}C. What is the temperature of the outside wall?

   Answer:
   We know q = KΔT/ΔL; ΔT = 10×0.16/0.04 = 40

   Therefore the outside temperature = -5+40 = 35^{o}C

3. A composite flat wall of a furnace is made up of two materials A and B. the thermal conductivity of A is twice that of material B, while the thickness of layer A is half that of B. if the temperature at the two sides of the wall are 400 and 1200 K, then what is the temperature drop across the layer of material A?

   Answer:

   $$\frac{800}{\frac{L/2}{2K}+\frac{L}{K}}=\frac{\Delta T}{\frac{L/2}{2K}}$$

   $$\Rightarrow \frac{800}{5L/4K}=\frac{\Delta T_1}{L/4K}$$

   $$\Rightarrow \Delta T_1 = 800/5 = 160K$$

4. For turbulent flow in a tube, the heat transfer coefficient is obtained from Dittus-Boilter correlation. If the tube diameter is halved and the flow rate is doubled, then the heat transfer co-efficient is changed by what factor?

   Answer:

   $$Nu = 0.023\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.33}$$

   $$\Rightarrow hD/K = (Dv\rho/\mu)^{0.8}(C_P\mu/K)^{0.33}$$

   $$\Rightarrow h \propto v^{0.8}D^{-0.2}$$

   Given that:

   $Q_2 = 2Q_1$ and $D_2 = D_1/2$

   From equation of continuity:

   $$Q \propto D^2 v$$

   $$Q_1 \propto D_1^2 v_1; Q_2 \propto D_2^2 v_2$$

   $$\Rightarrow 2D_1^2 v_1 = D_2^2 v_2 = (D_1/2)^2 v_2$$

   $$\Rightarrow v_2 = 8v_1$$

   Hence

   $$\frac{h_2}{h_1}=\frac{v_2^{0.8}D_2^{-0.2}}{v_1^{0.8}D_1^{-0.2}}=\frac{(8v_1)^{0.8}(D_1/2)^{-0.2}}{v_1^{0.8}D_1^{-0.2}}=8^{0.8}0.5^{-0.2}=6.06$$

5. The Sieder-Tate correlation for heat transfer in turbulent flow in a pipe gives $Nu \propto \mathrm{Re}^{0.8}$, where Nu is the Nusselt number and Re is the reynold's number for the flow. Assuming that this relation is valid, derive the relationship between heat transfer coefficient (h) and the pipe diameter (D).

   Answer:

   $$Nu \propto \mathrm{Re}^{0.8}$$

$$\Rightarrow \frac{hD}{K} \propto \left(\frac{Dv\rho}{\mu}\right)^{0.8}$$

$$\Rightarrow h \propto D^{-0.2}$$

6. The variation of thermal conductivity of a metal with temperature is often correlated using an expression of the form

   $k = k_0 + aT$

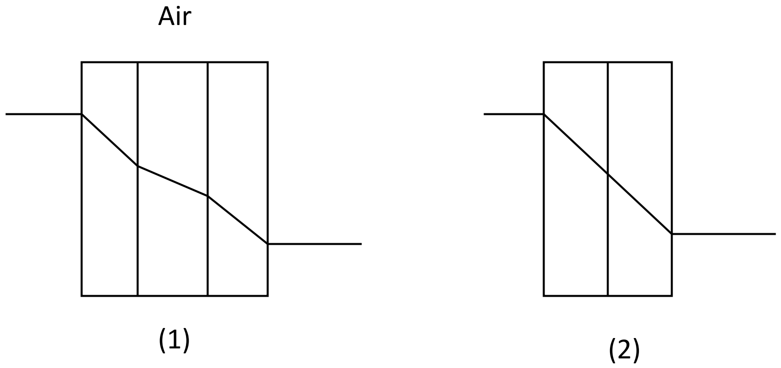
   Where k is the thermal conductivity, and T is the temperature in K. what is the units of ‘a’ in SI system?

   Answer: W/m. K

B. Long answer types

1. A thermopane window consists of two sheets of glass each 6 mm thick, separated by a layer of stagnant air also 6 mm thick. Find the percentage reduction in heat loss from this pane as compared to that of a single sheet of glass 6 mm thickness. The temperature drop between inside and outside remains same at 15 ^{o}C. thermal conductivity of glass is 30 times that of air.

   Solution:



$$q = \frac{\Delta T}{L/k}$$

Let k be the thermal conductivity of glass. Then thermal conductivity of air is k/30.

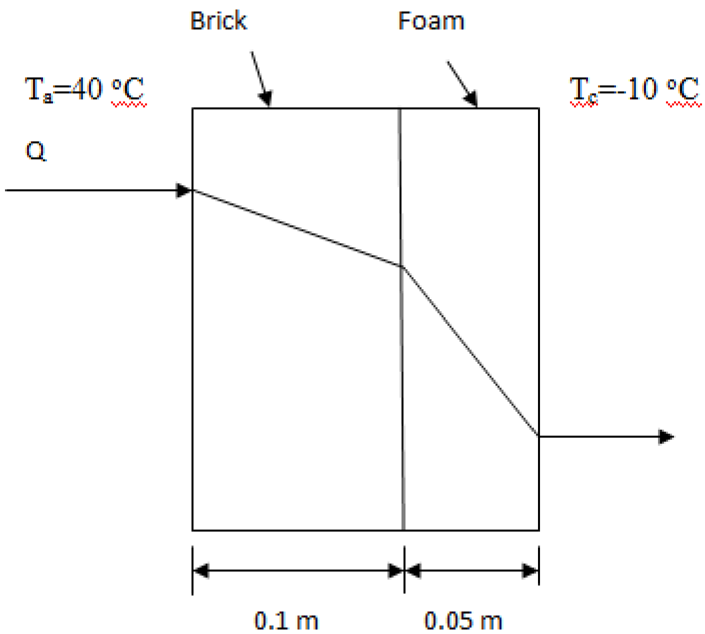
$$q_1 = \frac{15}{\frac{6}{k} + \frac{6\times 30}{k} + \frac{6}{k}} = \frac{15}{192}k$$

$$q_2 = \frac{15}{6/k} = \frac{15}{6}k$$

Reduction in heat loss = $\frac{q_2 - q_1}{q_2} \times 100 = \frac{(15/6) - (15/192)}{(15/6)} \times 100 = 96.9\%$

2. The wall of a cold storage unit comprises a brick layer (thickness $\delta_B = 0.1m$, thermal conductivity K_B = 1.4 W/m. K) and an inner layer of polyurethane foam (thickness δ_P = 0.05m, thermal conductivity K_B = 0.015 W/m. K). Assume one dimentional heat transfer by conduction through the composite wall, and that the inner surface of the polyurethane layer is at a temperature T_c and the outer surface of the brick layer at temperature T_h.

Solution:



Let T_i be the temperature at the contact of the two layers. Then

$$Q/A = q = \frac{k_b(T_h - T_i)}{\delta_B} = \frac{k_P(T_i - T_C)}{\delta_P}$$

By assuming the numerators and the denominators, heat flux q is obtained as

$$q = \frac{T_h - T_C}{(\delta_B/k_B) + (\delta_P/k_P)}$$

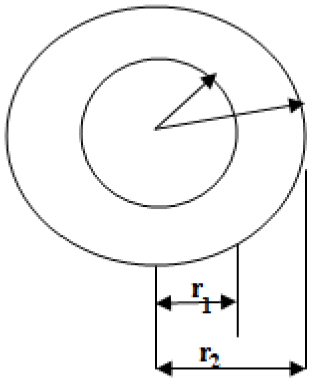
Rate of heat gain Q = qA

$$\Rightarrow Q = \frac{A(T_h - T_C)}{(\delta_B/k_B) + (\delta_P/k_P)}$$

$$\Rightarrow Q = \frac{260(40-(-10))}{(0.1/0.4)+(0.05/0.015)} = 3818W$$

3. The outside surface temperature of a pipe (radious = 0.1 m) is 400 K. The pipe is losing heat to atmosphere, which is at 300 K. the film heat transfer coefficient is 10 W/m2. K. To reduce the rate of heat loss, the pipe is insulated by a 50 mm thick layer of asbestos (k = 0.5 W/m. K). Calculate the percentage reduction in the rate of heat loss.

Solution:



Rate of heat loss without insulation (Q_1) = hAΔT

= 10×∏D×(400-300)

= 200π

Rate of heat loss with insulation (Q_2) = $\frac{\Delta T}{R_a + R_b}$

Where $R_a = \frac{1}{2\pi k}\ln\frac{r_1}{r_0} = \frac{1}{2\pi \times 0.5}\ln\left(\frac{0.1+0.05}{0.1}\right) = 0.4055/\pi$

$$R_b = \frac{1}{2\pi r_1 h} = \frac{1}{2\pi \times (0.1+0.05)\times 10} = 0.3333/\pi$$

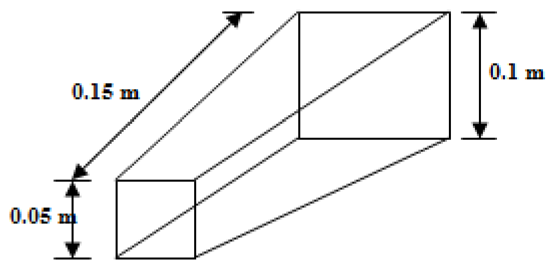
Therefore

$$Q_2 = \frac{400-300}{(0.4055+0.3333)/\pi} = 135.4\pi W/m$$

Reduction in heat loss = $\frac{Q_1 - Q_2}{Q_1} \times 100 = 32.3\%$

4. An asbestos pad, square in cross-section, measures 0.05 m on a side and increases linearly to 0.1 m on the side at the other end as shown in the figure. The length of the pad is 0.15

m. if the small end is held at 600 K and the larger end at 300 K. what will be the heat transfer flow rate if the other four sides are insulated. Assume one directional heat flow. Thermal conductivity of asbestos is 0.173 W/m. K.



Solution:

Variable area with respect to x:

$$A = 0.05\times 0.05 + x\left[\frac{0.1\times 0.1 - 0.05\times 0.05}{0.15}\right]$$

$$= 0.0025+0.05x$$

$$= 0.05(0.05+x)$$

Heat balance for the pad at a distance x from the small end, with thickness Δx,

$$[q_x A]_x - [q_x A]_{x+dx} = 0$$

Dividing by Δx, and in the limit as Δx→0,

$$\Rightarrow \frac{-d(Aq)}{dx} = 0$$

$$\Rightarrow Aq = constant = A_0 q_0$$

Where $A_0 = 0.05\times 0.05 = 0.0025\ m^2$

Writing the heat conduction equation,

$$Aq = -kA\frac{dT}{dx} = A_0 q_0$$

Substituting for A,

$$-k(0.05(0.05+x))\frac{dT}{dx} = 0.0025 q_0$$

Rearranging and integrating,

$$-\int_{600}^{300} dT = \frac{q_0 \times 0.0025}{0.05 \times 0.173} \int_0^{0.15} \frac{dx}{0.05 + x}$$

Solving, $q_0 = 1439\,\text{W/m}^2$

The heat flow rate = $A_0 q_0$ = 0.0025×1439 = 3.6 Watt

5. A liquid metal flows at a rate of 5 kg/sec through a 5 cm diameter stainless steel tube. It enters at 425 ^{o}C and is heated to 450 ^{o}C as it passes through the tube. If a constant heat flux is maintained along the tube and the tube wall is at a temperature 20 ^{o}C higher than the liquid metal bulk temperature, calculate the area required to affect the heat transfer.
   At constant heat flux,
   $Nu = 4.82 + 0.0185 Pe^{0.827}$
   Relation holds good.
   Properties of the compound:
   $\mu = 1.34 \times 10^{-3}$ kg/m. sec
   C_p = 0.149 kJ/kg. K
   K = 15.6 W/m. K
   Pr = 0.013

   Solution:
   Heat transfer rate Q,

   $Q = \dot{m} C_p \Delta T = 5 \times 0.149 \times (450 - 425) = 18.625\,\text{kW}$

   Estimation of heat transfer coefficient h:

   $$N_{\text{Re}} = \frac{Dv\rho}{\mu}$$

   Since, $v = \dfrac{\left(\dot{m}/\rho\right)}{(\pi/4)D^2}$

   Therefore, $v\rho = \dfrac{\dot{m}}{\pi/4D^2}$

   Hence, $\text{Re} = \dfrac{(0.05)(5)}{(1.34 \times 10^{-3})(\pi/4)(0.05)^2} = 95018$

   Pe = Re. Pr = 95018×0.013 = 1235.2

   Therefore from the given relation,

$$Nu = 4.82 + 0.0185\,Pe^{0.827} = 4.82 + 0.0185 \times (1235.2)^{0.827} = 11.49$$

Since $Nu = hD/k$,

$$h = \frac{kNu}{D} = \frac{15.6 \times 11.49}{0.05} = 3585\,\mathrm{W/\ m^2.\ K}$$

Estimation of heat transfer area A;

Rate of heat transfer by convection Q = hAΔT

Therefore, $A = \dfrac{Q}{h\Delta T} = \dfrac{18625}{3585 \times 20} = 0.26\,\mathrm{m^2}$

Area required to affect the heat transfer = 0.26 m^2