

INDIRA GANDHI INSTITUTE OF TECHNOLOGY SARANG

LECTURER NOTES

SUBJECT: **ADVANCED MASS TRANSFER**

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1.1 Introduction of Mass Transfer

When a system contains two or more components whose concentrations vary from point to point, there is a natural tendency for mass to be transferred, minimizing the concentration differences within a system. The transport of one constituent from a region of higher concentration to that of a lower concentration is called mass transfer.

The transfer of mass within a fluid mixture or across a phase boundary is a process that plays a major role in many industrial processes. Examples of such processes are:

- (i) Dispersion of gases from stacks
- (ii) Removal of pollutants from plant discharge streams by absorption
- (iii) Stripping of gases from waste water
- (iv) Neutron diffusion within nuclear reactors
- (v) Air conditioning

Many of our day-by-day experiences also involve mass transfer, for example:

- (i) A lump of sugar added to a cup of coffee eventually dissolves and then eventually diffuses to make the concentration uniform.
- (ii) Water evaporates from ponds to increase the humidity of passing-air-stream
- (iii) Perfumes presents a pleasant fragrance which is imparted throughout the surrounding atmosphere.

The mechanism of mass transfer involves both molecular diffusion and convection.

1.2 Properties of Mixtures

Mass transfer always involves mixtures. Consequently, we must account for the variation of physical properties which normally exist in a given system. When a system contains three or more components, as many industrial fluid streams do, the problem becomes unwieldy very quickly. The conventional engineering approach to problems of multicomponent system is to attempt to reduce them to representative binary (i.e., two component) systems.

In order to understand the future discussions, let us first consider definitions and relations which are often used to explain the role of components within a mixture.

1.2.1 Concentration of Species:

Concentration of species in multicomponent mixture can be expressed in many ways. For species A, mass concentration denoted by ρ_A is defined as the mass of A, m_A per unit volume of the mixture.

$$\rho_A = \frac{m_A}{V} \quad \text{-----} \quad (1)$$

The total mass concentration density ρ is the sum of the total mass of the mixture in unit volume:

$$\rho = \sum_i \rho_i$$

where ρ_i is the concentration of species i in the mixture.

Molar concentration of A, C_A is defined as the number of moles of A present per unit volume of the mixture.

By definition,

$$\text{Number of moles} = \frac{\text{mass of } A}{\text{molecular weight of } A}$$

$$n_A = \frac{m_A}{M_A} \quad \text{----- (2)}$$

Therefore from (1) & (2)

$$C_A = \frac{n_A}{V} = \frac{\rho_A}{M_A}$$

For ideal gas mixtures,

$$n_A = \frac{p_A V}{RT} \quad [\text{from Ideal gas law } PV = nRT]$$

$$C_A = \frac{n_A}{V} = \frac{p_A}{RT}$$

where p_A is the partial pressure of species A in the mixture. V is the volume of gas, T is the absolute temperature, and R is the universal gas constant.

The total molar concentration or molar density of the mixture is given by

$$C = \sum_i C_i$$

1.2.2 Velocities

In a multicomponent system the various species will normally move at different velocities; and evaluation of velocity of mixture requires the averaging of the velocities of each species present.

If v_i is the velocity of species i with respect to stationary fixed coordinates, then mass-average velocity for a multicomponent mixture defined in terms of mass concentration is,

$$v = \frac{\sum_i \rho_i v_i}{\sum_i \rho_i} = \frac{\sum_i \rho_i v_i}{\rho}$$

By similar way, molar-average velocity of the mixture v^* is

$$v^* = \frac{\sum_i C_i V_i}{C}$$

For most engineering problems, there will be little difference in v^* and v and so the mass average velocity, v , will be used in all further discussions.

The velocity of a particular species relative to the mass-average or molar average velocity is termed as diffusion velocity

(i.e.) Diffusion velocity = $v_i - v$

The mole fraction for liquid and solid mixture, x_A , and for gaseous mixtures, y_A , are the molar concentration of species A divided by the molar density of the mixtures.

$$x_A = \frac{C_A}{C} \quad (\text{liquids and solids})$$

$$y_A = \frac{C_A}{C} \quad (\text{gases}).$$

The sum of the mole fractions, by definition must equal 1;

$$\begin{aligned} \text{(i.e.)} \quad \sum_i x_i &= 1 \\ \sum_i y_i &= 1 \end{aligned}$$

by similar way, mass fraction of A in mixture is;

$$w_A = \frac{\rho_A}{\rho}$$

10. The molar composition of a gas mixture at 273 K and 1.5×10^5 Pa is:

O ₂	7%
CO	10%
CO ₂	15%
N ₂	68%

Determine

- a) the composition in weight percent
- b) average molecular weight of the gas mixture
- c) density of gas mixture
- d) partial pressure of O_2 .

Calculations:

Let the gas mixture constitutes 1 mole. Then

$$\begin{aligned}O_2 &= 0.07 \text{ mol} \\CO &= 0.10 \text{ mol} \\CO_2 &= 0.15 \text{ mol} \\N_2 &= 0.68 \text{ mol}\end{aligned}$$

Molecular weight of the constituents are:

$$\begin{aligned}O_2 &= 2 * 16 = 32 \text{ g/mol} \\CO &= 12 + 16 = 28 \text{ g/mol} \\CO_2 &= 12 + 2 * 16 = 44 \text{ g/mol} \\N_2 &= 2 * 14 = 28 \text{ g/mol}\end{aligned}$$

Weight of the constituents are: (1 mol of gas mixture)

$$\begin{aligned}O_2 &= 0.07 * 32 = 2.24 \text{ g} \\CO &= 0.10 * 28 = 2.80 \text{ g} \\CO_2 &= 0.15 * 44 = 6.60 \text{ g} \\N_2 &= 0.68 * 28 = 19.04 \text{ g}\end{aligned}$$

$$\begin{aligned}\text{Total weight of gas mixture} &= 2.24 + 2.80 + 6.60 + 19.04 \\&= 30.68 \text{ g}\end{aligned}$$

Composition in weight percent:

$$\begin{aligned}O_2 &= \frac{2.24}{30.68} * 100 = 7.30\% \\CO &= \frac{2.80}{30.68} * 100 = 9.13\% \\CO_2 &= \frac{6.60}{30.68} * 100 = 21.51\% \\N_2 &= \frac{19.04}{30.68} * 100 = 62.06\%\end{aligned}$$

Average molecular weight of the gas mixture $M = \frac{\text{Weight of gas mixture}}{\text{Number of moles}}$

$$M = \frac{30.68}{1} = 30.68 \text{ g/mol}$$

Assuming that the gas obeys ideal gas law,
 $PV = nRT$

$$\frac{n}{V} = \frac{P}{RT}$$

$$\frac{n}{V} = \text{molar density} = \rho_m$$

Therefore, density (or mass density) = $\rho_m M$
Where M is the molecular weight of the gas.

$$\begin{aligned} \text{Density} = \rho_m M &= \frac{PM}{RT} = \frac{1.5 * 10^5 * 30.68}{8314 * 273} \text{ kg/m}^3 \\ &= 2.03 \text{ kg/m}^3 \end{aligned}$$

Partial pressure of O_2 = [mole fraction of O_2] * total pressure

$$\begin{aligned} &= \frac{7}{100} * (1.5 * 10^5) \\ &= 0.07 * 1.5 * 10^5 \\ &= 0.105 * 10^5 \text{ Pa} \end{aligned}$$

1.3 Diffusion flux

Just as momentum and energy (heat) transfer have two mechanisms for transport-molecular and convective, so does mass transfer. However, there are convective fluxes in mass transfer, even on a molecular level. The reason for this is that in mass transfer, whenever there is a driving force, there is always a net movement of the mass of a particular species which results in a bulk motion of molecules. Of course, there can also be convective mass transport due to macroscopic fluid motion. In this chapter the focus is on molecular mass transfer.

The mass (or molar) flux of a given species is a vector quantity denoting the amount of the particular species, in either mass or molar units, that passes per given increment of time through a unit area normal to the vector. The flux of species defined with reference to fixed spatial coordinates, N_A is

$$N_A = C_A v_A \text{ ----- (1)}$$

This could be written in terms of diffusion velocity of A, (i.e., $v_A - v$) and average velocity of mixture, v , as

$$N_A = C_A (v_A - v) + C_A v \text{ ----- (2)}$$

By definition

$$v = v^* = \frac{\sum_i C_i v_i}{C}$$

Therefore, equation (2) becomes

$$\begin{aligned} N_A &= C_A (v_A - v) + \frac{C_A}{C} \sum_i C_i v_i \\ &= C_A (v_A - v) + y_A \sum_i C_i v_i \end{aligned}$$

For systems containing two components A and B,

$$\begin{aligned} N_A &= C_A (v_A - v) + y_A (C_A v_A + C_B v_B) \\ &= C_A (v_A - v) + y_A (N_A + N_B) \\ N_A &= C_A (v_A - v) + y_A N \text{ ----- (3)} \end{aligned}$$

The first term on the right hand side of this equation is diffusional molar flux of A, and the second term is flux due to bulk motion.

1.3.1 Fick's law:

An empirical relation for the diffusional molar flux, first postulated by Fick and, accordingly, often referred to as Fick's first law, defines the diffusion of component A in an isothermal, isobaric system. For diffusion in only the Z direction, the Fick's rate equation is

$$J_A = -D_{AB} \frac{dC_A}{dZ}$$

where D_{AB} is diffusivity or diffusion coefficient for component A diffusing through component B, and dC_A / dZ is the concentration gradient in the Z-direction.

A more general flux relation which is not restricted to isothermal, isobasic system could be written as

$$J_A = -C D_{AB} \frac{dy_A}{dz} \quad \text{----- (4)}$$

using this expression, Equation (3) could be written as

$$N_A = -C D_{AB} \frac{dy_A}{dz} + y_A N \quad \text{----- (5)}$$

1.3.2 Relation among molar fluxes:

For a binary system containing A and B, from Equation (5),

$$\begin{aligned} N_A &= J_A + y_A N \\ \text{or } J_A &= N_A + y_A N \quad \text{----- (6)} \end{aligned}$$

Similarly,

$$J_B = N_B + y_B N \quad \text{----- (7)}$$

Addition of Equation (6) & (7) gives,

$$J_A + J_B = N_A + N_B - (y_A + y_B) N \quad \text{----- (8)}$$

By definition $N = N_A + N_B$ and $y_A + y_B = 1$.

Therefore equation (8) becomes,

$$\begin{aligned} J_A + J_B &= 0 \\ J_A &= -J_B \end{aligned}$$

$$C D_{AB} \frac{dy_A}{dz} = -C D_{BA} \frac{dy_B}{dz} \quad \text{----- (9)}$$

$$\begin{aligned} \text{From } y_A + y_B &= 1 \\ dy_A &= -dy_B \end{aligned}$$

Therefore Equation (9) becomes,

$$D_{AB} = D_{BA} \quad \text{----- (10)}$$

This leads to the conclusion that diffusivity of A in B is equal to diffusivity of B in A.

1.4 Diffusivity

Fick's law proportionality, D_{AB} , is known as mass diffusivity (simply as diffusivity) or as the diffusion coefficient. D_{AB} has the dimension of L^2 / t , identical to the fundamental dimensions of the other transport properties: Kinematic viscosity, $\nu = (\mu / \rho)$ in momentum transfer, and thermal diffusivity, $\alpha (= k / \rho C_p)$ in heat transfer.

Diffusivity is normally reported in cm^2 / sec ; the SI unit being m^2 / sec .

Diffusivity depends on pressure, temperature, and composition of the system.

In table, some values of D_{AB} are given for a few gas, liquid, and solid systems.

Diffusivities of gases at low density are almost composition independent, increase with the temperature and vary inversely with pressure. Liquid and solid diffusivities are strongly concentration dependent and increase with temperature.

General range of values of diffusivity:

Gases :	5×10^{-6}	-----	1×10^{-5}	$m^2 / sec.$
Liquids :	10^{-6}	-----	10^{-9}	$m^2 / sec.$
Solids :	5×10^{-14}	-----	1×10^{-10}	$m^2 / sec.$

In the absence of experimental data, semi theoretical expressions have been developed which give approximation, sometimes as valid as experimental values, due to the difficulties encountered in experimental measurements.

1.4.1 Diffusivity in Gases:

Pressure dependence of diffusivity is given by

$$D_{AB} \propto \frac{1}{p} \quad (\text{for moderate ranges of pressures, upto 25 atm}).$$

And temperature dependency is according to

$$D_{AB} \propto T^{3/2}$$

Diffusivity of a component in a mixture of components can be calculated using the diffusivities for the various binary pairs involved in the mixture. The relation given by Wilke is

$$D_{1-mixture} = \frac{1}{\frac{y'_2}{D_{1-2}} + \frac{y'_3}{D_{1-3}} + \dots + \frac{y'_n}{D_{1-n}}}$$

Where $D_{1-mixture}$ is the diffusivity for component 1 in the gas mixture; D_{1-n} is the diffusivity for the binary pair, component 1 diffusing through component n; and y'_n is the mole fraction of component n in the gas mixture evaluated on a component –1 – free basis, that is

$$y'_2 = \frac{y_2}{y_2 + y_3 + \dots + y_n}$$

9. Determine the diffusivity of CO_2 (1), O_2 (2) and N_2 (3) in a gas mixture having the composition:

CO_2 : 28.5 %, O_2 : 15%, N_2 : 56.5%,

The gas mixture is at 273 K and 1.2×10^{-5} Pa. The binary diffusivity values are given as: (at 273 K)

$$D_{12} P = 1.874 \text{ m}^2 \text{ Pa/sec}$$

$$D_{13} P = 1.945 \text{ m}^2 \text{ Pa/sec}$$

$$D_{23} P = 1.834 \text{ m}^2 \text{ Pa/sec}$$

Calculations:

Diffusivity of CO_2 in mixture

$$D_{1m} = \frac{1}{\frac{y'_2}{D_{12}} + \frac{y'_3}{D_{13}}}$$

$$\text{where } y'_2 = \frac{y_2}{y_2 + y_3} = \frac{0.15}{0.15 + 0.565} = 0.21$$

$$y'_3 = \frac{y_3}{y_2 + y_3} = \frac{0.565}{0.15 + 0.565} = 0.79$$

$$\text{Therefore } D_{1m} P = \frac{1}{\frac{0.21}{1.874} + \frac{0.79}{1.945}} = 1.93 \text{ m}^2 \cdot \text{Pa/sec}$$

Since $P = 1.2 \times 10^5 \text{ Pa}$,

$$D_{1m} = \frac{1.93}{1.2 \times 10^5} = 1.61 \times 10^{-5} \text{ m}^2/\text{sec}$$

Diffusivity of O_2 in the mixture,

$$D_{2m} = \frac{1}{\frac{y_1'}{D_{21}} + \frac{y_3'}{D_{23}}}$$

$$\text{Where } y_1' = \frac{y_1}{y_1 + y_3} = \frac{0.285}{0.285 + 0.565} = 0.335$$

(mole fraction on-2 free bans).

and

$$y_3' = \frac{y_3}{y_1 + y_3} = \frac{0.565}{0.285 + 0.565} = 0.665$$

and

$$D_{21} P = D_{12} P = 1.874 \text{ m}^2 \cdot \text{Pa/sec}$$

Therefore

$$D_{2m} P = \frac{1}{\frac{0.335}{1.874} + \frac{0.665}{1.834}} = 1.847 \text{ m}^2 \cdot \text{Pa/sec}$$

$$D_{2m} = \frac{1.847}{1.2 \times 10^5} = 1.539 \times 10^{-5} \text{ m}^2/\text{sec}$$

By Similar calculations Diffusivity of N_2 in the mixture can be calculated, and is found to be, $D_{3m} = 1.588 \times 10^{-5} \text{ m}^2/\text{sec}$.

1.4.2 Diffusivity in liquids:

Diffusivity in liquid are exemplified by the values given in table ... Most of these values are nearer to $10^{-5} \text{ cm}^2 / \text{sec}$, and about ten thousand times slower than those in dilute gases. This characteristic of liquid diffusion often limits the overall rate of processes accruing in liquids (such as reaction between two components in liquids).

In chemistry, diffusivity limits the rate of acid-base reactions; in the chemical industry, diffusion is responsible for the rates of liquid-liquid extraction. Diffusion in liquids is important because it is slow.

Certain molecules diffuse as molecules, while others which are designated as electrolytes ionize in solutions and diffuse as ions. For example, sodium chloride (NaCl), diffuses in water as ions Na^+ and Cl^- . Though each ion has a different mobility, the electrical neutrality of the solution indicates the ions must diffuse at the same rate; accordingly it is possible to speak of a diffusion coefficient for molecular electrolytes such as NaCl . However, if several ions are present, the diffusion rates of the individual cations and anions must be considered, and molecular diffusion coefficients have no meaning.

Diffusivity varies inversely with viscosity when the ratio of solute to solvent ratio exceeds five. In extremely high viscosity materials, diffusion becomes independent of viscosity.

1.4.3 Diffusivity in solids:

Typical values for diffusivity in solids are shown in table. One outstanding characteristic of these values is their small size, usually thousands of times less than those in a liquid, which are in turn 10,000 times less than those in a gas.

Diffusion plays a major role in catalysis and is important to the chemical engineer. For metallurgists, diffusion of atoms within the solids is of more importance.

1.5 Steady State Diffusion

In this section, steady-state molecular mass transfer through simple systems in which the concentration and molar flux are functions of a single space coordinate will be considered.

In a binary system, containing A and B, this molar flux in the direction of z , as given by Eqn (5) is [section 1.3.1]

$$N_A = -CD_{AB} \frac{dy_A}{dz} + y_A (N_A + N_B) \quad \text{--- (1)}$$

1.5.1 Diffusion through a stagnant gas film

The diffusivity or diffusion coefficient for a gas can be measured, experimentally using Arnold diffusion cell.

The narrow tube of uniform cross section which is partially filled with pure liquid A, is maintained at a constant temperature and pressure. Gas B which flows across the open end of the tube, has a negligible solubility in liquid A, and is also chemically inert to A. (i.e. no reaction between A & B).

Component A vaporizes and diffuses into the gas phase; the rate of vaporization may be physically measured and may also be mathematically expressed in terms of the molar flux.

Consider the control volume $S \Delta z$, where S is the cross sectional area of the tube. Mass balance on A over this control volume for a steady-state operation yields

[Moles of A leaving at $z + \Delta z$] – [Moles of A entering at z] = 0.

$$(i.e.) \quad S N_A \Big|_{z+\Delta z} - S N_A \Big|_z = 0. \quad \text{-----} (1)$$

Dividing through by the volume, $S\Delta z$, and evaluating in the limit as Δz approaches zero, we obtain the differential equation

$$\frac{d N_A}{d z} = 0 \quad \text{-----} (2)$$

This relation stipulates a constant molar flux of A throughout the gas phase from z_1 to z_2 .

A similar differential equation could also be written for component B as,

$$\frac{d N_B}{d z} = 0,$$

and accordingly, the molar flux of B is also constant over the entire diffusion path from z_1 and z_2 .

Considering only at plane z_1 , and since the gas B is insoluble in liquid A, we realize that N_B , the net flux of B, is zero throughout the diffusion path; accordingly B is a stagnant gas.

From equation (1) (of section 1.5)

$$N_A = -CD_{AB} \frac{dy_A}{dz} + y_A (N_A + N_B)$$

Since $N_B = 0$,

$$N_A = -CD_{AB} \frac{dy_A}{dz} + y_A N_A$$

Rearranging,

$$N_A = \frac{-CD_{AB}}{1-y_A} \frac{dy_A}{dz} \quad \text{----- (3)}$$

This equation may be integrated between the two boundary conditions:

$$\begin{array}{ll} \text{at } z = z_1 & Y_A = Y_{A1} \\ \text{And at } z = z_2 & Y_A = y_{A2} \end{array}$$

Assuming the diffusivity is to be independent of concentration, and realizing that N_A is constant along the diffusion path, by integrating equation (3) we obtain

$$N_A \int_{z_1}^{z_2} dz = CD_{AB} \int_{y_{A1}}^{y_{A2}} \frac{-dy_A}{1-y_A}$$

$$N_A = \frac{CD_{AB}}{Z_2 - Z_1} \ln \left(\frac{1-y_{A2}}{1-y_{A1}} \right) \quad \text{----- (4)}$$

The log mean average concentration of component B is defined as

$$y_{B,lm} = \frac{y_{B2} - y_{B1}}{\ln \left(y_{B2} / y_{B1} \right)}$$

Since $y_B = 1 - y_A$,

$$y_{B,lm} = \frac{(1 - y_{A2}) - (1 - y_{A1})}{\ln\left(\frac{y_{A2}}{y_{A1}}\right)} = \frac{y_{A1} - y_{A2}}{\ln\left(\frac{y_{A2}}{y_{A1}}\right)} \quad \text{----- (5)}$$

Substituting from Eqn (5) in Eqn (4),

$$N_A = \frac{CD_{AB}}{Z_2 - Z_1} \frac{(y_{A1} - y_{A2})}{y_{B,lm}} \quad \text{----- (6)}$$

For an ideal gas $C = \frac{n}{V} = \frac{p}{RT}$, and

$$\text{for mixture of ideal gases } y_A = \frac{p_A}{P}$$

Therefore, for an ideal gas mixture equation. (6) becomes

$$N_A = \frac{D_{AB}}{RT(z_2 - z_1)} \frac{(p_{A1} - p_{A2})}{p_{B,lm}}$$

This is the equation of molar flux for steady state diffusion of one gas through a second stagnant gas.

Many mass-transfer operations involve the diffusion of one gas component through another non-diffusing component; absorption and humidification are typical operations defined by these equation.

12. Oxygen is diffusing in a mixture of oxygen-nitrogen at 1 std atm, 25°C. Concentration of oxygen at planes 2 mm apart are 10 and 20 volume % respectively. Nitrogen is non-diffusing.

- (a) Derive the appropriate expression to calculate the flux oxygen. Define units of each term clearly.
- (b) Calculate the flux of oxygen. Diffusivity of oxygen in nitrogen = $1.89 \times 10^{-5} \text{ m}^2/\text{sec}$.

Solution:

Let us denote oxygen as A and nitrogen as B. Flux of A (i.e.) N_A is made up of two components, namely that resulting from the bulk motion of A (i.e.), N_{x_A} and that resulting from molecular diffusion J_A :

$$N_A = Nx_A + J_A \text{ ----- (1)}$$

From Fick's law of diffusion,

$$J_A = -D_{AB} \frac{dC_A}{dz} \text{ ----- (2)}$$

Substituting this equation (1)

$$N_A = Nx_A - D_{AB} \frac{dC_A}{dz} \text{ ----- (3)}$$

Since $N = N_A + N_B$ and $x_A = C_A / C$ equation (3) becomes

$$N_A = (N_A + N_B) \frac{C_A}{C} - D_{AB} \frac{dC_A}{dz}$$

Rearranging the terms and integrating between the planes between 1 and 2,

$$\int \frac{dz}{CD_{AB}} = - \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{N_A C - C_A (N_A + N_B)} \text{ ----- (4)}$$

Since B is non diffusing $N_B = 0$. Also, the total concentration C remains constant. Therefore, equation (4) becomes

$$\begin{aligned} \frac{z}{CD_{AB}} &= - \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{N_A C - N_A C_A} \\ &= \frac{1}{N_A} \ln \frac{C - C_{A2}}{C - C_{A1}} \end{aligned}$$

Therefore,

$$N_A = \frac{CD_{AB}}{z} \ln \frac{C - C_{A2}}{C - C_{A1}} \text{ ----- (5)}$$

Replacing concentration in terms of pressures using Ideal gas law, equation (5) becomes

$$N_A = \frac{D_{AB} P_t}{RTz} \ln \frac{P_t - P_{A2}}{P_t - P_{A1}} \text{-----} (6)$$

where

D_{AB} = molecular diffusivity of A in B

P_T = total pressure of system

R = universal gas constant

T = temperature of system in absolute scale

z = distance between two planes across the direction of diffusion

P_{A1} = partial pressure of A at plane 1, and

P_{A2} = partial pressure of A at plane 2

Given:

$$D_{AB} = 1.89 \times 10^{-5} \text{ m}^2/\text{sec}$$

$$P_t = 1 \text{ atm} = 1.01325 \times 10^5 \text{ N/m}^2$$

$$T = 25^\circ\text{C} = 273 + 25 = 298 \text{ K}$$

$$z = 2 \text{ mm} = 0.002 \text{ m}$$

$$P_{A1} = 0.2 \times 1 = 0.2 \text{ atm (From Ideal gas law and additive pressure rule)}$$

$$P_{A2} = 0.1 \times 1 = 0.1 \text{ atm}$$

Substituting these in equation (6)

$$N_A = \frac{(1.89 \times 10^{-5})(1.01325 \times 10^5)}{(8314)(298)(0.002)} \ln \left(\frac{1 - 0.1}{1 - 0.2} \right)$$

$$= 4.55 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{sec}$$

1.5.2 Psuedo steady state diffusion through a stagnant film:

In many mass transfer operations, one of the boundaries may move with time. If the length of the diffusion path changes a small amount over a long period of time, a pseudo steady state diffusion model may be used. When this condition exists, the equation of steady state diffusion through stagnant gas' can be used to find the flux.

If the difference in the level of liquid A over the time interval considered is only a small fraction of the total diffusion path, and $t_0 - t$ is relatively long period of time, at any given instant in that period, the molar flux in the gas phase may be evaluated by

$$N_A = \frac{C D_{AB} (y_{A1} - y_{A2})}{z y_{B,lm}} \text{----- (1)}$$

where z equals $z_2 - z_1$, the length of the diffusion path at time t .

The molar flux N_A is related to the amount of A leaving the liquid by

$$N_A = \frac{\rho_{A,L}}{M_A} \frac{dz}{dt} \text{----- (2)}$$

where $\frac{\rho_{A,L}}{M_A}$ is the molar density of A in the liquid phase

under Psuedo steady state conditions, equations (1) & (2) can be equated to give

$$\frac{\rho_{A,L}}{M_A} \frac{dz}{dt} = \frac{C D_{AB} (y_{A1} - y_{A2})}{z y_{B,lm}} \text{----- (3)}$$

Equation. (3) may be integrated from $t = 0$ to t and from $z = z_{t0}$ to $z = z_t$ as:

$$\int_{t=0}^t dt = \frac{\rho_{A,L} y_{B,lm} / M_A}{C D_{AB} (y_{A1} - y_{A2})} \int_{z_{t0}}^{z_t} z dz$$

yielding

$$t = \frac{\rho_{A,L} y_{B,lm} / M_A}{C D_{AB} (y_{A1} - y_{A2})} \left(\frac{z_t^2 - z_{t0}^2}{2} \right) \text{----- (4)}$$

This shall be rearranged to evaluate diffusivity D_{AB} as,

$$D_{AB} = \frac{\rho_{A,L} y_{B,lm}}{M_A C (y_{A1} - y_{A2}) t} \left(\frac{z_t^2 - z_{t0}^2}{2} \right)$$

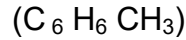
1. A vertical glass tube 3 mm in diameter is filled with liquid toluene to a depth of 20mm from the top openend. After 275 hrs at 39.4 °C and a total pressure of 760 mm Hg the level has dropped to 80 mm from the top. Calculate the value of diffusivity.

Data:

vapor pressure of toluene at $39.4^{\circ}\text{C} = 7.64 \text{ kN} / \text{m}^2$,

density of liquid toluene = 850 kg/m^3

Molecular weight of toluene = 92



$$D_{AB} = \frac{\rho_{A,L} y_{B,lm}}{M_A C (y_{A1} - y_{A2}) t} \left(\frac{Z_t^2 - Z_{t0}^2}{2} \right)$$

$$\text{where } y_{B,lm} = \frac{y_{B2} - y_{B1}}{\ln \left(\frac{y_{B2}}{y_{B1}} \right)}$$

$$y_{B2} = 1 - y_{A2}$$

$$y_{B1} = 1 - y_{A1}$$

$$y_{A1} = \frac{p_{A1}}{P} = \frac{7.64}{101.3} \quad (760 \text{ mm Hg} = 101.3 \text{ kN/m}^2)$$

$$= 0.0754$$

$$y_{B1} = 1 - 0.0754 = 0.9246$$

$$y_{A2} = 0$$

$$y_B = 1 - y_{A2} = 1$$

$$\text{Therefore } y_{B,lm} = \frac{1 - 0.9246}{\ln \left(\frac{1}{0.9246} \right)} = 0.9618$$

$$C = \frac{P}{RT} = \frac{1.01325 \times 10^5}{8314 \times (273 + 39.4)}$$

$$= 0.039 \text{ k mol /m}^3$$

Therefore

$$D_{AB} = \frac{850 \times 0.9618}{92 \times 0.039 \times (0.0754 - 0) \times 275 \times 3600} \left(\frac{0.08^2 - 0.02^2}{2} \right)$$

$$= 1.5262 \times 10^{-3} (0.08^2 - 0.02^2)$$

$$= 9.1572 \times 10^{-6} \text{ m}^2/\text{sec.}$$

1.5.3 Equimolar counter diffusion:

A physical situation which is encountered in the distillation of two constituents whose molar latent heats of vaporization are essentially equal, stipulates that the flux of one gaseous component is equal to but acting in the opposite direction from the other gaseous component; that is, $N_A = - N_B$.

The molar flux N_A , for a binary system at constant temperature and pressure is described by

$$N_A = -C D_{AB} \frac{dy_A}{dz} + y_A (N_A + N_B)$$

or
$$N_A = -D_{AB} \frac{dC_A}{dz} + y_A (N_A + N_B) \quad \text{----- (1)}$$

with the substitution of $N_B = -N_A$, Equation (1) becomes,

$$N_A = -D_{AB} \frac{dC_A}{dz} \quad \text{----- (2)}$$

For steady state diffusion Equation. (2) may be integrated, using the boundary conditions:

$$\begin{array}{ll} \text{at } z = z_1 & C_A = C_{A1} \\ \text{and } z = z_2 & C_A = C_{A2} \end{array}$$

Giving,

$$N_A \int_{z_1}^{z_2} dz = -D_{AB} \int_{C_{A1}}^{C_{A2}} dC_A$$

from which

$$N_A = \frac{D_{AB}}{z_2 - z_1} (C_{A1} - C_{A2}) \quad \text{----- (3)}$$

For ideal gases, $C_A = \frac{n_A}{V} = \frac{p_A}{RT}$. Therefore Equation. (3) becomes

$$N_A = \frac{D_{AB}}{RT(z_2 - z_1)} (P_{A1} - P_{A2}) \quad \text{----- (4)}$$

This is the equation of molar flux for steady-state equimolar counter diffusion.

Concentration profile in these equimolar counter diffusion may be obtained from,

$$\frac{d}{dz} (N_A) = 0 \quad (\text{Since } N_A \text{ is constant over the diffusion path}).$$

And from equation. (2)

$$N_A = -D_{AB} \frac{dC_A}{dz}.$$

Therefore

$$\frac{d}{dz} \left(-D_{AB} \frac{dC_A}{dz} \right) = 0.$$

or
$$\frac{d^2 C_A}{dz^2} = 0.$$

This equation may be solved using the boundary conditions to give

$$\frac{C_A - C_{A1}}{C_{A1} - C_{A2}} = \frac{z - z_1}{z_1 - z_2} \quad \text{----- (5)}$$

Equation, (5) indicates a linear concentration profile for equimolar counter diffusion.

3. Methane diffuses at steady state through a tube containing helium. At point 1 the partial pressure of methane is $p_{A1} = 55$ kPa and at point 2, 0.03 m apart $p_{A2} = 15$ kPa. The total pressure is 101.32 kPa, and the temperature is 298 K. At this pressure and temperature, the value of diffusivity is $6.75 \times 10^{-5} \text{ m}^2/\text{sec}$.

- i) calculate the flux of CH_4 at steady state for equimolar counter diffusion.
- ii) Calculate the partial pressure at a point 0.02 m apart from point 1.

Calculation:

For steady state equimolar counter diffusion, molar flux is given by

$$N_A = \frac{D_{AB}}{RTz} (p_{A1} - p_{A2}) \quad \text{----- (1)}$$

Therefore;

$$N_A = \frac{6.75 \times 10^{-5}}{8.314 \times 298 \times 0.03} (55 - 15) \frac{\text{kmol}}{\text{m}^2 \cdot \text{sec}}$$

$$= 3.633 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \text{ sec}}$$

And from (1), partial pressure at 0.02 m from point 1 is:

$$3.633 \times 10^{-5} = \frac{6.75 \times 10^{-5}}{8.314 \times 298 \times 0.02} (55 - p_A)$$

$$p_A = 28.33 \text{ kPa}$$

11. In a gas mixture of hydrogen and oxygen, steady state equimolar counter diffusion is occurring at a total pressure of 100 kPa and temperature of 20°C. If the partial pressures of oxygen at two planes 0.01 m apart, and perpendicular to the direction of diffusion are 15 kPa and 5 kPa, respectively and the mass diffusion flux of oxygen in the mixture is $1.6 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{sec}$, calculate the molecular diffusivity for the system.

Solution:

For equimolar counter current diffusion:

$$N_A = \frac{D_{AB}}{RTz} (p_{A1} - p_{A2}) \text{----- (1)}$$

where

N_A = molar flux of A ($1.6 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{sec}$):

D_{AB} = molecular diffusivity of A in B

R = Universal gas constant (8.314 kJ/kmol.k)

T = Temperature in absolute scale (273 + 20 = 293 K)

z = distance between two measurement planes 1 and 2 (0.01 m)

P_{A1} = partial pressure of A at plane 1 (15 kPa); and

P_{A2} = partial pressure of A at plane 2 (5 kPa)

Substituting these in equation (1)

$$1.6 \times 10^{-5} = \frac{D_{AB}}{(8.314)(293)(0.01)} (15 - 5)$$

Therefore, $D_{AB} = 3.898 \times 10^{-5} \text{ m}^2/\text{sec}$

2. A tube 1 cm in inside diameter that is 20 cm long is filled with CO_2 and H_2 at a total pressure of 2 atm at 0°C . The diffusion coefficient of the $\text{CO}_2 - \text{H}_2$ system under these conditions is $0.275 \text{ cm}^2/\text{sec}$. If the partial pressure of CO_2 is 1.5 atm at one end of the tube and 0.5 atm at the other end, find the rate of diffusion for:

- i) steady state equimolar counter diffusion ($N_A = -N_B$)
- ii) steady state counter diffusion where $N_B = -0.75 N_A$, and
- iii) steady state diffusion of CO_2 through stagnant H_2 ($N_B = 0$)

$$\text{i) } N_A = -C D_{AB} \frac{d y_A}{d z} + y_A (N_A + N_B)$$

Given

$$N_B = -N_A$$

$$\text{Therefore } N_A = -C D_{AB} \frac{d y_A}{d z} = -D_{AB} \frac{d C_A}{d z}$$

(For ideal gas mixture $C_A = \frac{p_A}{RT}$ where p_A is the partial pressure of A; such that $p_A + p_B = P$)

$$\text{Therefore } N_A = -D_{AB} \frac{d (p_A/RT)}{d z}$$

For isothermal system, T is constant

$$\text{Therefore } N_A = \frac{-D_{AB}}{RT} \frac{d p_A}{d z}$$

$$\text{(i.e.) } N_A \int_{Z_1}^{Z_2} d z = -\frac{D_{AB}}{RT} \int_{P_{A1}}^{P_{A2}} d p_A$$

$$N_A = \frac{D_{AB}}{RT Z} (p_{A1} - p_{A2}) \text{ ----- (1)}$$

where $Z = Z_2 - Z_1$

Given: $D_{AB} = 0.275 \text{ cm}^2/\text{sec} = 0.275 \times 10^{-4} \text{ m}^2/\text{sec}$; $T = 0^\circ\text{C} = 273 \text{ K}$

$$N_A = \frac{0.275 * 10^{-4}}{8314 * 273 * 0.2} \left(1.5 * 1.01325 * 10^5 - 0.5 * 1.01325 * 10^5 \right)$$

$$= 6.138 * 10^{-6} \frac{k \text{ mol}}{m^2 \text{ sec}}$$

Rate of diffusion = $N_A S$

Where S is surface area

$$\begin{aligned} \text{Therefore rate of diffusion} &= 6.138 * 10^{-6} * \pi r^2 \\ &= 6.138 * 10^{-6} * \pi (0.5 * 10^{-2})^2 \\ &= 4.821 * 10^{-10} \text{ k mol/sec} \\ &= 1.735 * 10^{-3} \text{ mol/hr.} \end{aligned}$$

$$\text{ii) } N_A = -C D_{AB} \frac{d y_A}{d z} + y_A (N_A + N_B)$$

$$\text{given: } N_B = -0.75 N_A$$

$$\text{Therefore } N_A = -C D_{AB} \frac{d y_A}{d z} + y_A (N_A - 0.75 N_A)$$

$$= -C D_{AB} \frac{d y_A}{d z} + 0.25 y_A N_A$$

$$N_A - 0.25 y_A N_A = -C D_{AB} \frac{d y_A}{d z}$$

$$N_A d z = -C D_{AB} \frac{d y_A}{1 - 0.25 y_A}$$

for constant N_A and C

$$N_A \int_{Z_1}^{Z_2} d z = -C D_{AB} \int_{y_{A1}}^{y_{A2}} \frac{d y_A}{1 - 0.25 y_A}$$

$$\left[\int \frac{d x}{a + b x} = \frac{1}{b} \ln(a + b x) \right]$$

$$N_A z = (-C D_{AB}) \left(\frac{-1}{0.25} \right) \left[\ln(1 - 0.25 y_A) \right]_{y_{A1}}^{y_{A2}}$$

$$N_A = -\frac{4CD_{AB}}{z} \ln \left(\frac{1-0.25y_{A2}}{1-0.25y_{A1}} \right) \text{----- (2)}$$

Given:

$$C = \frac{p}{RT} = \frac{2 * 1.01325 * 10^5}{8314 * 273} = 0.0893 \text{ K mol/m}^3$$

$$y_{A1} = \frac{p_{A1}}{P} = \frac{1.5}{2} = 0.75$$

$$y_{A2} = \frac{p_{A2}}{P} = \frac{0.5}{2} = 0.25$$

Substituting these in equation (2),

$$\begin{aligned} N_A &= \frac{4 * 0.0893 * 0.275 * 10^{-4}}{0.2} \left[\ln \frac{1 - 0.25 * 0.25}{1 - 0.25 * 0.75} \right] \\ &= 7.028 * 10^{-6} \frac{\text{kmol}}{\text{m}^2 \text{ sec}} \end{aligned}$$

$$\begin{aligned} \text{Rate of diffusion} = N_A S &= 7.028 * 10^{-6} * \pi * (0.5 * 10^{-2})^2 \\ &= 5.52 * 10^{-10} \text{ kmol/sec} \\ &= 1.987 * 10^{-3} \text{ mol/hr.} \end{aligned}$$

$$\text{iii) } N_A = -CD_{AB} \frac{dy_A}{dz} + y_A (N_A + N_B)$$

Given: $N_B = 0$

$$\text{Therefore } N_A = -CD_{AB} \frac{dy_A}{dz} + y_A N_A$$

$$\begin{aligned} N_A \int_{Z_1}^{Z_2} dz &= -CD_{AB} \int_{y_{A1}}^{y_{A2}} \frac{dy_A}{1-y_A} \\ &= \frac{CD_{AB}}{Z} \ln \left(\frac{1-y_{A2}}{1-y_{A1}} \right) \\ &= \frac{0.0893 * 0.275 * 10^{-4}}{0.2} \left[\ln \left(\frac{1-0.25}{1-0.75} \right) \right] \end{aligned}$$

$$= 1.349 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{sec}}$$

$$\text{Rate of diffusion} = 1.349 \times 10^{-5} \times \pi \times (0.5 \times 10^{-2})^2$$

$$\begin{aligned} &= 1.059 \text{ Kmol / sec} \\ &= 3.814 \text{ mol/hr} \end{aligned}$$

1.5.4 Diffusion into an infinite standard medium:

Here we will discuss problems involving diffusion from a spherical particle into an infinite body of stagnant gas. The purpose in doing this is to demonstrate how to set up differential equations that describe the diffusion in these processes. The solutions, obtained are only of academic interest because a large body of gas in which there are no convection currents is unlikely to be found in practice. However, the solutions developed here for these problems actually represent a special case of the more common situation involving both molecular diffusion and convective mass transfer.

a) Evaporation of a spherical Droplet:

As an example of such problems, we shall consider the evaporation of spherical droplet such as a raindrop or sublimation of naphthalene ball. The vapor formed at the surface of the droplet is assumed to diffuse by molecular motions into the large body of stagnant gas that surrounds the droplet.

Consider a raindrop, at any moment, when the radius of the drop is r_0 , the flux of water vapor at any distance r from the center is given by

$$N_A = -C D_{AB} \frac{dy_A}{dr} + y_A (N_A + N_B)$$

Here $N_B = 0$ (since air is assumed to be stagnant)
Therefore,

$$N_A = -C D_{AB} \frac{dy_A}{dr} + y_A N_A$$

Rearranging,

$$N_A = \frac{-C D_{AB}}{1 - y_A} \frac{dy_A}{dr} \quad \text{_____} \quad (1)$$

The flux N_A is not constant, because of the spherical geometry; decreases as the distance from the center of sphere increases. But the molar flow rate at r and $r + \delta r$ are the same.

This could be written as,

$$A N_A \Big|_r = A N_A \Big|_{r + \delta r} \quad \text{_____} \quad (2)$$

where A = surface area of sphere at r or $r + \delta r$.

Substituting for $A = 4 \pi r^2$ in equation (2),

$$4\pi r^2 N_A \Big|_{r+\delta r} - 4\pi r^2 N_A \Big|_r = 0$$

or

$$\lim_{\delta r \rightarrow 0} \frac{r^2 N_A \Big|_{r+\delta r} - r^2 N_A \Big|_r}{\delta r} = 0$$

$$\frac{d}{dr} (r^2 N_A) = 0 \quad \text{_____ (3)}$$

Integrating,

$$r^2 N_A = \text{constant} \quad \text{_____ (4)}$$

From equation (4), $r^2 N_A = r_0^2 N_{A0}$

Substituting for N_A from equation (1),

$$\frac{-r^2 C D_{AB}}{1-y_A} \frac{dy_A}{dr} = r_0^2 N_{A0}$$

$$r_0^2 N_{A0} \int \frac{dr}{r^2} = -C D_{AB} \int \frac{dy_A}{1-y_A} \quad \text{_____ (5)}$$

Boundary condition :

$$\text{At } r = r_0 \quad y_A = y_{AS}$$

And

$$\text{At } r = \infty \quad y_A = y_{A\infty}$$

Therefore equation (5) becomes,

$$r_0^2 N_{A0} \left(-\frac{1}{r} \right)_{r_0}^{\infty} = [C D_{AB} \ln(1-y_A)]_{y_{AS}}^{y_{A\infty}}$$

Simplifying,

$$N_{A0} = \frac{C D_{AB}}{r_0} \ln \left(\frac{1-y_{A\infty}}{1-y_{AS}} \right) \quad \text{_____ (6)}$$

Time required for complete evaporation of the droplet may be evaluated from making mass balance.

$$\frac{\text{Moles of water diffusing}}{\text{unit time}} = \frac{\text{moles of water leaving the droplet}}{\text{unit time}}$$

$$4\pi r_0^2 N_{A0} = -\frac{d}{dt} \left(\frac{4}{3} \pi r_0^3 \frac{\rho_L}{M_A} \right)$$

$$= -4\pi r_0^2 \frac{\rho_L}{M_A} \frac{dr_0}{dt} \quad \text{_____} \quad (7)$$

Substituting for N_{A0} from equation (6) in equation (7),

$$\frac{CD_{AB}}{r_0} \ln\left(\frac{1-y_A}{1-y_{AS}}\right) = \frac{-\rho_L}{M_A} \frac{dr_0}{dt} \quad \text{_____} \quad (8)$$

Initial condition :

When $t = 0$ $r_0 = r_1$

Integrating equation (8) with these initial condition,

$$\int_0^t dt = \frac{-\rho_L}{M_A} \frac{1}{CD_{AB}} \frac{1}{\ln\left(\frac{1-y_{A\infty}}{1-y_{AS}}\right)} \int_{r_1}^0 r_0 dr_0$$

$$t = \frac{\rho_L}{M_A} \frac{1}{2CD_{AB}} \frac{r_1^2}{\ln\left(\frac{1-y_{A\infty}}{1-y_{AS}}\right)} \quad \text{_____} \quad (9)$$

Equation (9) gives the total time t required for complete evaporation of spherical droplet of initial radius r_1 .

b) Combustion of a coal particle:

The problem of combustion of spherical coal particle is similar to evaporation of a drop with the exception that chemical reaction (combustions) occurs at the surface of the particle. During combustion of coal, the reaction



occurs. According to this reaction for every mole of oxygen that diffuses to the surface of coal (maximum of carbon), react with 1 mole of carbon, releases 1 mole of carbon dioxide, which must diffuse away from this surface. This is a case of equimolar counter diffusion of CO_2 and O_2 . Normally air (a mixture of N_2 and O_2) is used for combustion, and in this case N_2 does not takes part in the reaction, and its flux is zero. (i.e. $N_{N_2} = 0$).

The molar flux of O_2 could be written as

$$N_{O_2} = -CD_{O_2-gas} \frac{dy_{O_2}}{dr} + y_{O_2} (N_{O_2} + N_{CO_2} + N_{N_2}) \quad \text{_____} \quad (1)$$

Where D_{O_2-gas} is the diffusivity of O_2 in the gas mixture.

Since $N_{N_2} = 0$, and from stoichiometry $N_{O_2} = -N_{CO_2}$, equation (1) becomes

$$N_{O_2} = -C D_{O_2-gas} \frac{d y_{O_2}}{d r} \quad \text{_____} \quad (2)$$

For steady state conditions,

$$\frac{d}{d r} (r^2 N_{O_2}) = 0 \quad \text{_____} \quad (3)$$

Integrating,

$$r^2 N_{O_2} = \text{constant} = r_0^2 N_{O_2 s} \quad \text{_____} \quad (4)$$

Where r_0 is the radius of coal particle at any instant, and $N_{O_2 s}$ is the flux of O_2 at the surface of the particle.

Substituting for N_{O_2} from equation (2) in equation (4),

$$-r^2 C D_{O_2-gas} \frac{d y_{O_2}}{d r} = r_0^2 N_{O_2 s} \quad \text{_____} \quad (5)$$

Boundary condition :

$$\text{At } r = r_0 \quad y_{O_2} = y_{O_2 s}$$

And

$$\text{At } r = \infty \quad y_{O_2} = y_{O_2 \infty}$$

With these boundary condition, equation (5) becomes

$$r_0^2 N_{A_0} \int_{r_0}^{\infty} \frac{d r}{r^2} = -C D_{O_2-gas} \int_{y_{O_2 s}}^{y_{O_2 \infty}} d y_{O_2}$$

which yields

$$N_{O_2 s} = \frac{C D_{O_2-gas}}{r_0} (y_{O_2 s} - y_{O_2 \infty}) \quad \text{_____} \quad (6)$$

For fast reaction of O_2 with coal, the mole fraction of O_2 at the surface of particle is zero. (i.e.,) $y_{O_2 s} = 0$.

And also at some distance away from the surface of the particle $y_{O_2} = y_{O_2 \infty} = 0.21$ (because air is a mixture of 21 mole % O_2 and 79 mole % N_2)

With these conditions, equation (6) becomes,

$$N_{O_2 s} = \frac{0.21 C D_{O_2-gas}}{r_0} \quad \text{_____} \quad (7)$$

5. A sphere of naphthalene having a radius of 2mm is suspended in a large volume of shell air at 318 K and 1 atm. The surface pressure of the naphthalene

can be assumed to be at 318 K is 0.555 mm Hg. The D_{AB} of naphthalene in air at 318 K is $6.92 \times 10^{-6} \text{ m}^2/\text{sec}$. Calculate the rate of evaporation of naphthalene from the surface.

Calculation

Steady state mass balance over a element of radius r and $r + \delta r$ leads to

$$S N_A \Big|_r - S N_A \Big|_{r + \delta r} = 0 \text{ ----- (1)}$$

where S is the surface area ($= 4 \pi r^2$)

dividing (1) by $S \delta r$, and taking the limit as δr approaches zero, gives:

$$\frac{d(r^2 N_A)}{dr} = 0$$

Integrating $r^2 N_A = \text{constant}$ (or) $4 \pi r^2 N_A = \text{constant}$

We can assume that there is a film of naphthalene – vapor / air film around naphthalene through which molecular diffusion occurs.

Diffusion of naphthalene vapor across this film could be written as,

$$N_A = -CD_{AB} \frac{dy_A}{dr} + y_A (N_A + N_B)$$

$N_B = 0$ (since air is assumed to be stagnant in the film)

$$N_A = -CD_{AB} \frac{dy_A}{dr} + y_A N_A$$

$$N_A = -CD_{AB} \frac{d}{dr} \left(\frac{y_A}{1 - y_A} \right)$$

$$N_A = CD_{AB} \frac{d[\ln(1 - y_A)]}{dr}$$

$W_A = \text{Rate of evaporation} = 4 \pi r^2 N_A \Big|_R = \text{constant.}$

$$W_A = \frac{4 \pi r^2 CD_{AB} d(\ln(1 - y_A))}{dr}$$

$$W_A \int \frac{dr}{r^2} = 4\pi D_{AB} \int C d \ln(1 - y_A)$$

Boundary condition:

$$\begin{aligned} \text{At } r = R \quad y_A &= \frac{0.555}{760} = 7.303 \times 10^{-4} \\ \ln(1 - y_A) &= -7.3 \times 10^{-4} \\ \text{At } r = \infty \quad y_A &= 0 \quad \ln(1 - y_A) = 0 \end{aligned}$$

$$\text{Therefore } W_A \int_R^\infty \frac{dr}{r^2} = 4\pi D_{AB} C \int_{-7.3 \times 10^{-4}}^0 d[\ln(1 - y_A)]$$

$$W_A \left[\frac{-1}{r} \right]_R^\infty = 4\pi D_{AB} C [\ln(1 - y_A)]_{-7.3 \times 10^{-4}}^0$$

$$W_A \left[0 + \frac{1}{R} \right] = 4\pi D_{AB} C [0 + 7.3 \times 10^{-4}]$$

$$W_A = 4\pi R D_{AB} C * 7.3 \times 10^{-4}$$

$$C = \frac{P}{\text{Gas constant} * T} = \frac{1.01325 * 10^5}{8314 * 318} = 0.0383 \text{ kmol/m}^3$$

Initial rate of evaporation:

$$\begin{aligned} \text{Therefore } W_A &= 4 * 3.142 * 2 * 10^{-3} * 6.92 * 10^{-6} * 0.0383 * 7.3 * 10^{-4} \\ &= 4.863 * 10^{-12} \text{ kmol/sec} \\ &= 1.751 * 10^{-5} \text{ mol/hr.} \end{aligned}$$

1.5.5 Diffusion in Liquids:

Equation derived for diffusion in gases equally applies to diffusion in liquids with some modifications. Mole fraction in liquid phases is normally written as 'x' (in gases as y). The concentration term 'C' is replaced by average molar density, $\left(\frac{\rho}{M}\right)_{av}$.

a) For steady – state diffusion of A through non diffusivity B:

$$N_A = \text{constant}, N_B = 0$$

$$N_A = \frac{D_{AB}}{Z x_{BM}} \left(\frac{\rho}{M}\right)_{av} (x_{A1} - x_{A2})$$

where $Z = Z_2 - Z_1$, the length of diffusion path; and

$$X_{BM} = \frac{X_{B2} - X_{B1}}{\ln\left(\frac{X_{B2}}{X_{B1}}\right)}$$

b) For steady – state equimolar counter diffusion :

$$N_A = -N_B = \text{const}$$

$$N_A = \frac{D_{AB}}{Z} (C_{A1} - C_{A2}) = \frac{D_{AB}}{Z} \left(\frac{\rho}{M}\right)_{av} (x_{A1} - x_{A2})$$

4. Calculate the rate of diffusion of butanol at 20°C under unidirectional steady state conditions through a 0.1 cm thick film of water when the concentrations of butanol at the opposite sides of the film are, respectively 10% and 4% butanol by weight. The diffusivity of butanol in water solution is $5.9 \times 10^{-6} \text{ cm}^2/\text{sec}$. The densities of 10% and 4% butanol solutions at 20°C may be taken as 0.971 and 0.992 g/cc respectively. Molecular weight of Butanol ($\text{C}_4\text{H}_9\text{OH}$) is 74, and that of water 18.

Calculations

For steady state unidirectional diffusion,

$$N_A = \frac{D_{AB}}{Z} C \frac{(x_{A1} - x_{A2})}{x_{B,lm}}$$

where C is the average molar density.

$$= \left(\frac{\rho}{M}\right)_{avg}$$

Conversion from weight fraction the Mole fraction:

$$x_{A1} = \frac{(0.1/74)}{(0.1/74 + 0.9/18)} = 0.026$$

$$x_{A2} = \frac{(0.04/74)}{(0.04/74 + 0.96/18)} = 0.010$$

Average molecular weight at 1 & 2:

$$M_1 = \frac{1}{(0.1/74 + 0.9/18)} = 19.47 \text{ kg/Kmol}$$

$$M_2 = \frac{1}{(0.04/74 + 0.96/18)} = 18.56 \text{ kg/Kmol}$$

$$\begin{aligned}\left(\frac{\rho}{M}\right)_{avg} &= \frac{(\rho_1/M_1 + \rho_2/M_2)}{2} \\ &= \frac{0.971/19.47 + 0.992/18.56}{2} \\ &= 0.0517 \text{ gmol / cm}^3 \\ &= 51.7 \text{ kmol/m}^3\end{aligned}$$

$$x_{B,lm} = \frac{x_{B2} - x_{B1}}{\ln(x_{B2}/x_{B1})} = \frac{(1 - x_{A2}) - (1 - x_{A1})}{\ln\left(\frac{1 - x_{A2}}{1 - x_{A1}}\right)}$$

$$\begin{aligned}\text{(i.e.) } x_{B,lm} &= \frac{(1 - 0.01) - (1 - 0.026)}{\ln\left(\frac{1 - 0.01}{1 - 0.026}\right)} \\ &= \frac{0.016}{0.0163} = 0.982\end{aligned}$$

$$\begin{aligned}\text{Therefore } N_A &= \frac{D_{AB}}{2} \left(\frac{\rho}{M}\right)_{avg} \frac{(x_{A1} - x_{A2})}{x_{B,lm}} \\ &= \frac{5.9 * 10^{-6} * 10^{-4} * 51.7}{0.1 * 10^{-2}} * \frac{(0.026 - 0.010)}{0.982} \\ &= 4.97 * 10^{-7} \frac{\text{kmol}}{\text{m}^2 \text{ sec}} \\ &= 1.789 \frac{\text{gmol}}{\text{m}^2 \cdot \text{hr.}} \\ &= 1.789 * 74 \frac{\text{g}}{\text{m}^2 \cdot \text{hr.}} \\ &= 132.4 \frac{\text{g}}{\text{m}^2 \cdot \text{hr.}}\end{aligned}$$

Mass diffusion with homogeneous chemical reaction:

Absorption operations involves contact of a gas mixture with a liquid and preferential dissolution of a component in the contacting liquid. Depending on the chemical nature of the involved molecules, the absorption may or may not involve chemical reaction.

The following analysis illustrates the diffusion of a component from the gas phase into the liquid phase accompanied by a chemical reaction in the liquid phase. Consider a layer of absorbing medium (liquid).

At the surface of the liquid, the composition of A is C_{A0} . The thickness of the film, δ is so defined, that beyond this film the concentration of A is always zero ; that is $C_{A\delta} = 0$. If there is very little fluid motion within the film,

$$N_A = -D_{AB} \frac{dC_A}{dz} + \frac{C_A}{C} (N_A + N_B) \quad \text{_____} \quad (1)$$

If concentration of A in the film, C_A is assumed small, equation (1) becomes

$$N_A = -D_{AB} \frac{dC_A}{dz} \quad \text{_____} \quad (2)$$

The molar flux N_A changes along the diffusion path. This change is due to the reaction that takes place in the liquid film. This changes could be written as

$$\frac{d}{dz} (N_A) - r_A = 0 \quad \text{_____} \quad (3)$$

where $-r_A$ is the rate of disappearance of A. For a first order reaction,



with the substitution from equation (4) and (2) in equation (3),

$$\frac{-d}{dz} \left(D_{AB} \frac{dC_A}{dz} \right) + k C_A = 0$$

For constant Diffusivity,

$$-D_{AB} \frac{d^2 C_A}{dz^2} + k C_A = 0 \quad \text{_____} \quad (5)$$

which is a second order ordinary differential equation. The general solution to this equation is

$$C_A = C_1 \cosh \left(\sqrt{k/D_{AB}} z \right) + C_2 \sinh \left(\sqrt{k/D_{AB}} z \right) \quad \text{_____} \quad (6)$$

The constants of this equation can be evaluated from the boundary conditions:

at $Z = 0$ $C_A = C_{A0}$
 And at $Z = \delta$ $C_A = 0$.

The constant C_1 is equal to C_{A0} , and C_2 is equal to $-C_{A0} / \tanh \left(\sqrt{k/D_{AB}} \delta \right)$ with this substitution equation (6) becomes,

$$C_A = C_{A0} \cosh \left(\sqrt{k/D_{AB}} z \right) - \frac{C_{A0} \sinh \left(\sqrt{k/D_{AB}} z \right)}{\tanh \left(\sqrt{k/D_{AB}} \delta \right)} \quad (7)$$

This equation gives the variation of concentration of A with z (i.e. concentration profile of A in the liquid). The molar flux at the liquid surface can be determined by differentiating equation (7), and evaluating the derivative

$$dC_A/dz \text{ at } z = 0$$

Differentiating C_A with respect to z,

$$\frac{dC_A}{dz} = C_{A0} \sqrt{k/D_{AB}} \sinh \left(\sqrt{k/D_{AB}} z \right) - \frac{C_{A0} \sqrt{k/D_{AB}} \cosh \left(\sqrt{k/D_{AB}} z \right)}{\tanh \left(\sqrt{k/D_{AB}} \delta \right)} \quad (8)$$

Substituting $z = 0$ in equation (8) and from equation (2),

$$N_A|_{z=0} = \frac{D_{AB} C_{A0}}{\delta} \left[\frac{\sqrt{k/D_{AB}} \delta}{\tanh \left(\sqrt{k/D_{AB}} \delta \right)} \right] \quad (9)$$

For absorption with no chemical reaction, the flux of A is obtained from equation (2) as

$$N_A = \frac{D_{AB} C_{A0}}{\delta} \quad (10)$$

which is constant throughout the film of liquid. On comparison of equation (9) and

(10), it is apparent that the term $\left[\sqrt{k/D_{AB}} \delta / \left(\tanh \left(\sqrt{k/D_{AB}} \delta \right) \right) \right]$ shows

the influence of the chemical reactions. This term is a dimensionless quantity, is often called as Hatta Number.

1.5.7 Diffusion in solids

In certain unit operation of chemical engineering such as in drying or in absorption, mass transfer takes place between a solid and a fluid phase. If the transferred species is distributed uniformly in the solid phase and forms a homogeneous medium, the diffusion of the species in the solid phase is said to

be structure independent. In this cases diffusivity or diffusion coefficient is direction – independent.

At steady state, and for mass diffusion which is independent of the solid matrix structure, the molar flux in the z direction is :

$$N_A = -D_{AB} \frac{dC_A}{dz} = \text{constant}, \text{ as given by Fick's law.}$$

Integrating the above equation,

$$N_A = \frac{D_{AB} (C_{A1} - C_{A2})}{z}$$

which is similar to the expression obtained for diffusion in a stagnant fluid with no bulk motion (i.e. $N = 0$).

Diffusion in process solids:

In some chemical operations, such as heterogeneous catalysis, an important factor, affecting the rate of reaction is the diffusions of the gaseous component through a porous solid. The effective diffusivity in the solid is reduced below what it could be in a free fluid, for two reasons. First, the tortuous nature of the path increases the distance, which a molecule must travel to advance a given distance in the solid. Second, the free cross – sectional area is restricted. For many catalyst pellets, the effective diffusivity of a gaseous component is of the order of one tenth of its value in a free gas.

If the pressure is low enough and the pores are small enough, the gas molecules will collide with the walls more frequently than with each other. This is known as Knudsen flow or Knudsen diffusion. Upon hitting the wall, the molecules are momentarily absorbed and then given off in random directions. The gas flux is reduced by the wall collisions.

By use of the kinetic flux is the concentration gradient is independent of pressure ; whereas the proportionality constant for molecular diffusion in gases (i.e. Diffusivity) is inversely proportional to pressure.

Knudsen diffusion occurs when the size of the pore is of the order of the mean free path of the diffusing molecule.

1.6 Transient Diffusion

Transient processes, in which the concentration at a given point varies with time, are referred to as unsteady state processes or time – dependent processes. This variation in concentration is associated with a variation in the mass flux.

These generally fall into two categories:

- i) the process which is in an unsteady state only during its initial startup, and
- ii) the process which is in a batch operation throughout its operation.

In unsteady state processes there are three variables-concentration, time, and position. Therefore the diffusion process must be described by partial rather than ordinary differential equations.

Although the differential equations for unsteady state diffusion are easy to establish, most solutions to these equations have been limited to situations involving simple geometries and boundary conditions, and a constant diffusion coefficient.

Many solutions are for one-directional mass transfer as defined by Fick's second law of diffusion :

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2} \quad \text{_____} \quad (1)$$

This partial differential equation describes a physical situation in which there is no bulk-motion contribution, and there is no chemical reaction. This situation is encountered when the diffusion takes place in solids, in stationary liquids, or in system having equimolar counter diffusion. Due to the extremely slow rate of diffusion within liquids, the bulk motion contribution of flux equation (i.e., $y_A \sum N_i$) approaches the value of zero for dilute solutions ; accordingly this system also satisfies Fick's second law of diffusion.

The solution to Fick's second law usually has one of the two standard forms. It may appear in the form of a trigonometric series which converges for large values of time, or it may involve series of error functions or related integrals which are most suitable for numerical evaluation at small values of time. These solutions are commonly obtained by using the mathematical techniques of separation of variables or Laplace transforms.

Convective Mass Transfer

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2.2 Convective Mass Transfer coefficient

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2.8.2 Wet bulb thermometer

2.1 Introduction

Our discussion of mass transfer in the previous chapter was limited to molecular diffusion, which is a process resulting from a concentration gradient. In system involving liquids or gases, however, it is very difficult to eliminate convection from the overall mass-transfer process.

Mass transfer by convection involves the transport of material between a boundary surface (such as solid or liquid surface) and a moving fluid or between two relatively immiscible, moving fluids.

There are two different cases of convective mass transfer:

1. Mass transfer takes place only in a single phase either to or from a phase boundary, as in sublimation of naphthalene (solid form) into the moving air.
2. Mass transfer takes place in the two contacting phases as in extraction and absorption.

In the first few section we will see equation governing convective mass transfer in a single fluid phase.

2.2 Convective Mass Transfer Coefficient

In the study of convective heat transfer, the heat flux is connected to heat transfer coefficient as

$$Q/A = q = h(t_s - t_m) \text{ ----- (4.1)}$$

The analogous situation in mass transfer is handled by an equation of the form

$$N_A = k_c (C_{As} - C_A) \text{ ----- (4.2)}$$

The molar flux N_A is measured relative to a set of axes fixed in space. The driving force is the difference between the concentration at the phase boundary, C_{As} (a solid surface or a fluid interface) and the concentration at some arbitrarily defined point in the fluid medium, C_A . The convective mass transfer coefficient k_c is a function of geometry of the system and the velocity and properties of the fluid similar to the heat transfer coefficient, h .

2.3 Significant Parameters in Convective Mass Transfer

Dimensionless parameters are often used to correlate convective transfer data. In momentum transfer Reynolds number and friction factor play a major role. In the correlation of convective heat transfer data, Prandtl and Nusselt numbers are important. Some of the same parameters, along with some newly defined dimensionless numbers, will be useful in the correlation of convective mass-transfer data.

The molecular diffusivities of the three transport process (momentum, heat and mass) have been defined as:

$$\text{Momentum diffusivity } \nu = \frac{\mu}{\rho} \text{ ----- (4.3)}$$

$$\text{Thermal diffusivity } \alpha = \frac{k}{\rho C_p} \text{ ----- (4.4)}$$

and

$$\text{Mass diffusivity } D_{AB} \text{ ----- (4.5)}$$

It can be shown that each of the diffusivities has the dimensions of L^2 / t , hence, a ratio of any of the two of these must be dimensionless.

The ratio of the molecular diffusivity of momentum to the molecular diffusivity of heat (thermal diffusivity) is designated as the Prandtl Number

$$\frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \text{Pr} = \frac{\nu}{\alpha} = \frac{C_p \mu}{K} \text{ ----- (4.6)}$$

The analogous number in mass transfer is Schmidt number given as

$$\frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} = \text{Sc} = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}} \text{ ----- (4.7)}$$

The ratio of the molecular diffusivity of heat to the molecular diffusivity of mass is designated the Lewis Number, and is given by

$$\frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = \text{Le} = \frac{\alpha}{D_{AB}} = \frac{k}{\rho C_p D_{AB}} \text{ ----- (4.8)}$$

Lewis number is encountered in processes involving simultaneous convective transfer of mass and energy.

Let us consider the mass transfer of solute A from a solid to a fluid flowing past the surface of the solid. For such a case, the mass transfer between the solid surface and the fluid may be written as

$$N_A = k_c (C_{As} - C_{A\infty}) \text{ ----- (4.1 a)}$$

Since the mass transfer at the surface is by molecular diffusion, the mass transfer may also be described by

$$N_A = -D_{AB} \left. \frac{dC_A}{dy} \right|_{y=0} \text{ ----- (4.9)}$$

When the boundary concentration, C_{As} is constant, equation (4.9) may be written as

$$N_A = -D_{AB} \left. \frac{d(C_A - C_{As})}{dy} \right|_{y=0} \text{ ----- (4.10)}$$

Equation (4.1a) and (4.10) may be equated, since they define the same flux of component A leaving the surface and entering the fluid

$$k_c (C_{As} - C_{A\infty}) = -D_{AB} \left. \frac{d(C_A - C_{As})}{dy} \right|_{y=0} \text{ ----- (4.11)}$$

This relation may be rearranged into the following form:

$$\frac{k_c}{D_{AB}} = - \left. \frac{d(C_A - C_{As})/dy}{(C_A - C_{A\infty})} \right|_{y=0} \text{ ----- (4.12)}$$

Multiplying both sides of equation (4.12) by a characteristic length, L we obtain the following dimensionless expression:

$$\frac{k_c L}{D_{AB}} = - \frac{d(C_A - C_{As})/dy \big|_{y=0}}{(C_{As} - C_{A\infty})/L} \text{ ----- (4.13)}$$

The right hand side of equation (4.13) is the ratio of the concentration gradient at the surface to an overall or reference concentration gradient; accordingly, it may be considered as the ratio of molecular mass-transport resistance to the convective mass-transport resistance of the fluid. This ratio is generally known as the Sherwood number, Sh and analogous to the Nusselt number Nu, in heat transfer.

2.4 Application of Dimensionless Analysis to Mass Transfer

One of the method of obtaining equations for predicting mass-transfer coefficients is the use of dimensionless analysis. Dimensional analysis predicts the various dimensionless parameters which are helpful in correlating experimental data.

There are two important mass transfer processes, which we shall consider, the transfer of mass into a stream flowing under forced convection and the transfer of mass into a phase which is moving as the result of natural convection associated with density gradients.

2.4.1 Transfer into a stream flowing under forced convection

Consider the transfer of mass from the walls of a circular conduit to a fluid flowing through the conduit. The mass transfer is due to the concentration driving force $C_{As} - C_A$.

These variables include terms descriptive of the system geometry, the flow and fluid properties and the quantity of importance, k_c .

By the Buckingham method of grouping the variables, the number of dimensionless π groups is equal to the number of variables minus the number of fundamental dimensions. Hence the number of dimensionless group for this problem will be three.

With D_{AB} , ρ and D as the core variables, the three π groups to be formed are

$$\pi_1 = D_{AB}^a \rho^b D^c k_c \text{ ----- (4.14)}$$

$$\pi_2 = D_{AB}^d \rho^e D^f g \text{ ----- (4.15)}$$

$$\text{and } \pi_3 = D_{AB}^g \rho^h D^i \mu \text{ ----- (4.16)}$$

Substituting the dimensions for π ,

$$\pi_1 = D_{AB}^a \rho^b D^c k_c \text{ ----- (4.17)}$$

$$1 = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \left(\frac{L}{t}\right) \text{ ----- (4.18)}$$

Equating the exponents of the fundamental dimensions on both sides of the equation, we have

$$L : \quad 0 = 2a - 3b + c + 1$$

$$t : \quad 0 = -a - 1$$

$$M : \quad 0 = b$$

Solving these equations,

$$a = -1, \quad b = 0 \quad \text{and} \quad c = 1$$

Thus $\pi_1 = \frac{k_c D}{D_{AB}}$ which is the Sherwood number.

The other two π groups could be determined in the same manner, yielding

$$\pi_2 = \frac{D \nu}{D_{AB}} \text{ ----- (4.19)}$$

$$\text{and } \pi_3 = \frac{\mu}{\rho D_{AB}} = S_c \text{ ----- (4.20)}$$

which is termed as Schmidt Number

Dividing π_2 by π_3 , we get

$$\frac{\pi_2}{\pi_3} = \left(\frac{D \nu}{D_{AB}}\right) \bigg/ \left(\frac{\mu}{\rho D_{AB}}\right) = \frac{D \nu \rho}{\mu} = \text{Re} \text{ ----- (4.21)}$$

which is the Reynolds Number

The result of the dimensional analysis of mass transfer by forced convection in a circular conduit indicates that a correlating relation could be of the form,

$$Sh = \psi (Re, Sc) \text{ ----- (4.22)}$$

Which is analogous to the heat transfer correlation

$$Nu = \psi (Re, Pr) \text{ ----- (4.23)}$$

2.4.2 Transfer into a phase whose motion is due to Natural Convection

Natural convection currents develop if there exists any variation in density within the fluid phase. The density variation may be due to temperature differences or to relatively large concentration differences.

According to Buckingham theorem, there will be three dimensionless groups. Choosing D_{AB} , L and μ as the core variables, the π groups to be formed are

$$\pi_1 = D_{AB}^a L^b \mu^c k_c \text{ ----- (4.24)}$$

$$\pi_2 = D_{AB}^d L^e \mu^f \rho \text{ ----- (4.25)}$$

and $\pi_3 = D_{AB}^g L^h \mu^i g \Delta \rho_A \text{ ----- (4.26)}$

Solving for the dimensionless groups, we obtain

$$\pi_1 = \frac{k_c L}{D_{AB}} = Nu, \text{ the Nusselt number ----- (4.27)}$$

$$\pi_2 = \frac{\rho D_{AB}}{\mu} = \frac{1}{Sc}, \text{ the reciprocal of Schmidt number ----- (4.28)}$$

and $\pi_3 = \frac{L^3 g \Delta \rho_A}{\mu D_{AB}} \text{ ----- (4.29)}$

With the multiplication of π_2 and π_3 , we obtain a dimensionless parameter analogous to the Grashof number in heat transfer by natural convection

$$\pi_2 \pi_3 = \left(\frac{\rho D_{AB}}{\mu} \right) \left(\frac{L^3 g \Delta \rho_A}{\mu D_{AB}} \right)$$

$$= \frac{L^3 \rho g \Delta \rho_A}{\mu^2} = Gr_{AB} \text{ ----- (4.30)}$$

The result of the dimensional analysis of mass transfer by natural convection indicates that a correlating relation could be of the form,

$$Sh = \psi (Gr_{AB}, Sc) \text{ ----- (4.31)}$$

2.5 Analysis among Mass, Heat and Momentum Transfer

Analogies among mass, heat and momentum transfer have their origin either in the mathematical description of the effects or in the physical parameters used for quantitative description.

To explore those analogies, it could be understood that the diffusion of mass and conduction of heat obey very similar equations. In particular, diffusion in one dimension is described by the Fick's Law as

$$J_A = -D_{AB} \frac{dC_A}{dz} \text{ ----- (4.32)}$$

Similarly, heat conduction is described by Fourier's law as

$$q = -k \frac{dT}{dz} \text{ ----- (4.33)}$$

Where k is the thermal conductivity.

The similar equation describing momentum transfer as given by Newton's law is

$$\tau = -\mu \frac{dv}{dz} \text{ ----- (4.34)}$$

Where τ is the momentum flux (or shear stress) and μ is the viscosity of fluid.

At this point it has become conventional to draw an analogy among mass, heat and momentum transfer. Each process uses a simple law combined with a mass or energy or momentum balance.

In this section, we shall consider several analogies among transfer phenomenon which has been proposed because of the similarity in their mechanisms. The analogies are useful in understanding the transfer phenomena and as a satisfactory means for predicting behaviour of systems for which limited quantitative data are available.

The similarity among the transfer phenomena and accordingly the existence of the analogies require that the following five conditions exist within the system

1. The physical properties are constant
2. There is no mass or energy produced within the system. This implies that there is no chemical reaction within the system
3. There is no emission or absorption of radiant energy.
4. There is no viscous dissipation of energy.
5. The velocity profile is not affected by the mass transfer. This implies there should be a low rate of mass transfer.

2.5.1 Reynolds Analogy

The first recognition of the analogous behaviour of mass, heat and momentum transfer was reported by Osborne Reynolds in 1874. Although his analogy is limited in application, it served as the base for seeking better analogies.

Reynolds postulated that the mechanisms for transfer of momentum, energy and mass are identical. Accordingly,

$$\frac{k_c}{v_\infty} = \frac{h}{\rho v_\infty C_p} = \frac{f}{2} \text{-----} (4.35)$$

Here h is heat transfer coefficient

f is friction factor

v_∞ is velocity of free stream

The Reynolds analogy is interesting because it suggests a very simple relation between different transport phenomena. This relation is found to be accurate when Prandtl and Schmidt numbers are equal to one. This is applicable for mass transfer by means of turbulent eddies in gases. In this situation, we can estimate mass transfer coefficients from heat transfer coefficients or from friction factors.

2.5.2 Chilton – Colburn Analogy

Because the Reynold's analogy was practically useful, many authors tried to extend it to liquids. Chilton and Colburn, using experimental data, sought modifications to the Reynold's analogy that would not have the restrictions that Prandtl and Schmidt numbers must be equal to one. They defined for the j factor for mass transfer as

$$j_D = \frac{k_c}{v_\infty} (Sc)^{2/3} \text{ ----- (4.36)}$$

The analogous j factor for heat transfer is

$$j_H = St Pr^{2/3} \text{ ----- (4.37)}$$

where St is Stanton number = $\frac{Nu}{Re Pr} = \frac{h}{\rho g_\infty C_p}$

Based on data collected in both laminar and turbulent flow regimes, they found

$$j_D = j_H = \frac{f}{2} \text{ ----- (4.38)}$$

This analogy is valid for gases and liquids within the range of $0.6 < Sc < 2500$ and $0.6 < Pr < 100$.

The Chilton-Colburn analogy has been observed to hold for many different geometries for example, flow over flat plates, flow in pipes, and flow around cylinders.

13. A stream of air at 100 kPa pressure and 300 K is flowing on the top surface of a thin flat sheet of solid naphthalene of length 0.2 m with a velocity of 20 m/sec. The other data are:

Mass diffusivity of naphthalene vapor in air = $6 \times 10^{-6} \text{ m}^2/\text{sec}$

Kinematic viscosity of air = $1.5 \times 10^{-5} \text{ m}^2/\text{sec}$

Concentration of naphthalene at the air-solid naphthalene interface = $1 \times 10^{-5} \text{ kmol/m}^3$

Calculate:

- (a) the overage mass transfer coefficient over the flat plate
- (b) the rate of loss of naphthalene from the surface per unit width

Note: For heat transfer over a flat plate, convective heat transfer coefficient for laminar flow can be calculated by the equation.

$$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$$

you may use analogy between mass and heat transfer.

Solution:

Given: Correlation for heat transfer

$$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$$

The analogous relation for mass transfer is

$$Sh = 0.664 Re_L^{1/2} Sc^{1/3} \text{ -----(1)}$$

where

Sh = Sherwood number = kL/D_{AB}
 Re_L = Reynolds number = $Lv\rho/\mu$
 Sc = Schmidt number = $\mu / (\rho D_{AB})$
 k = overall mass transfer coefficient
 L = length of sheet
 D_{AB} = diffusivity of A in B
 v = velocity of air
 μ = viscosity of air
 ρ = density of air, and
 μ/ρ = kinematic viscosity of air.

Substituting for the known quantities in equation (1)

$$\frac{k(0.2)}{6 \times 10^{-6}} = 0.664 \left(\frac{(0.2)(20)}{1.5 \times 10^{-5}} \right)^{1/2} \left(\frac{1.5 \times 10^{-5}}{6 \times 10^{-6}} \right)^{1/3}$$

$k = 0.014 \text{ m/sec}$

$$\begin{aligned} \text{Rate of loss of naphthalene} &= k (C_{Ai} - C_{A\infty}) \\ &= 0.014 (1 \times 10^{-5} - 0) = 1.4024 \times 10^{-7} \text{ kmol/m}^2 \text{ sec} \end{aligned}$$

$$\begin{aligned} \text{Rate of loss per meter width} &= (1.4024 \times 10^{-7}) (0.2) = 2.8048 \times 10^{-8} \text{ kmol/m.sec} \\ &= 0.101 \text{ gmol/m.hr.} \end{aligned}$$

2.6 Convective Mass Transfer Correlations

Extensive data have been obtained for the transfer of mass between a moving fluid and certain shapes, such as flat plates, spheres and cylinders. The techniques include sublimation of a solid, vapourization of a liquid into a moving stream of air and the dissolution of a solid into water.

These data have been correlated in terms of dimensionless parameters and the equations obtained are used to estimate the mass transfer coefficients in other moving fluids and geometrically similar surfaces.

2.6.1 Flat Plate

From the experimental measurements of rate of evaporation from a liquid surface or from the sublimation rate of a volatile solid surface into a controlled air-stream, several correlations are available. These correlation have been found to satisfy the equations obtained by theoretical analysis on boundary layers,

$$Sh = 0.664 Re_L^{1/2} Sc^{1/3} \text{ (laminar) } Re_L < 3 * 10^5 \quad \text{-----} \quad (4.39)$$

$$Sh = 0.036 Re_L^{0.8} Sc^{1/3} \text{ (turbulent) } Re_L > 3 * 10^5 \quad \text{-----} \quad (4.40)$$

Using the definition of j factor for mass transfer on equation (4.39) and (4.40) we obtain

$$j_D = 0.664 Re_L^{-1/2} \text{ (laminar) } Re_L < 3 * 10^5 \quad \text{-----} \quad (4.41)$$

$$j_D = 0.037 Re_L^{-0.2} \text{ (turbulent) } Re_L > 3 * 10^5 \quad \text{-----} \quad (4.42)$$

These equations may be used if the Schmidt number in the range $0.6 < Sc < 2500$.

7. If the local Nusselt number for the laminar boundary layer that is formed over a flat plate is

$$Nu_x = 0.332 Re_x^{1/2} Sc^{1/3}$$

Obtain an expression for the average film-transfer coefficient $\bar{k}_c$, when the Reynolds number for the plate is

- a) $Re_L = 100\ 000$
- b) $Re_L = 1500\ 000$

The transition from laminar to turbulent flow occurs at $Re_x = 3 \times 10^5$.

Derivation:

$$\text{By definition : } \bar{k}_c = \frac{\int_0^L k_c dx}{\int_0^L dx}$$

$$\text{and } Nu_x = \frac{k_c x}{D_{AB}}; \quad Re_x = \frac{x v \rho}{\mu}; \quad Sc = \frac{\mu}{\rho D_{AB}};$$

For $Re_L = 100\,000$; (which is less than the Reynolds number corresponding to Transition value of 3×10^5)

$$\begin{aligned} \bar{k}_c &= \frac{\int_0^L 0.332 \left(\frac{x v \rho}{\mu} \right)^{\frac{1}{2}} (Sc)^{\frac{1}{3}} \frac{D_{AB}}{x} dx}{L} \\ &= \frac{0.332 (Sc)^{1/3} \left(\frac{v \rho}{\mu} \right)^{1/2}}{L} D_{AB} \int_0^L \frac{dx}{x^{1/2}} \\ &= \frac{0.332}{\frac{1}{2} L} Sc^{1/3} \left(\frac{v \rho}{\mu} \right)^{1/2} D_{AB} \left[x^{1/2} \right]_0^L \end{aligned}$$

$$\text{(i.e.) } \frac{\bar{k}_c L}{D_{AB}} = 0.664 Re_L^{1/2} Sc^{1/3} \quad [\text{answer (a)}]$$

For $Re_L = 1500\,000 (> 3 \times 10^5)$

$$\bar{k}_c = D_{AB} \frac{\left(\int_0^{L_t} 0.332 Re_x^{1/2} Sc^{1/3} \frac{dx}{x} + \int_{L_t}^L 0.0292 Re_x^{4/5} Sc^{1/3} \frac{dx}{x} \right)}{L}$$

where L_t is the distance from the leading edge of the plane to the transition point where $Re_x = 3 \times 10^5$.

$$\bar{k}_c = D_{AB} \frac{\left(0.332 Sc^{1/3} \left(\frac{v \rho}{\mu} \right)^{1/2} \int_0^{L_t} \frac{dx}{x^{1/2}} + 0.0292 Sc^{1/3} \left(\frac{v \rho}{\mu} \right)^{4/5} \int_{L_t}^L \frac{dx}{x^{1/5}} \right)}{L}$$

$$\begin{aligned} \frac{\bar{k}_c L}{D_{AB}} &= 0.664 Re_t^{1/2} Sc^{1/3} + \frac{0.0292}{4/5} Sc^{1/3} \left[x^{4/5} \right]_{L_t}^L \left(\frac{V \rho}{\mu} \right)^{4/5} \\ &= 0.664 Re_t^{1/2} Sc^{1/3} + 0.0365 Sc^{1/3} \left(Re_L^{4/5} - Re_t^{4/5} \right) \\ \frac{\bar{k}_c L}{D_{AB}} &= 0.664 Re_t^{1/2} Sc^{1/3} + 0.0365 Re_L^{4/5} Sc^{1/3} - 0.0365 Re_t^{4/5} Sc^{1/3} \end{aligned}$$

where $Re_t = 3 \times 10^5$

2.6.2 Single Sphere

Correlations for mass transfer from single spheres are represented as addition of terms representing transfer by purely molecular diffusion and transfer by forced convection, in the form

$$Sh = Sh_o + C Re^m Sc^n \text{ ----- (4.43)}$$

Where C, m and n are constants, the value of n is normally taken as 1/3. For very low Reynold's number, the Sherwood number should approach a value of 2. This value has been derived in earlier sections by theoretical consideration of molecular diffusion from a sphere into a large volume of stagnant fluid. Therefore the generalized equation becomes

$$Sh = 2 + C Re^m Sc^{1/3} \text{ ----- (4.44)}$$

For mass transfer into liquid streams, the equation given by Brain and Hales

$$Sh = \left(4 + 1.21 Pe_{AB}^{2/3} \right)^{1/2} \text{ ----- (4.45)}$$

correlates the data that are obtained when the mass transfer Peclet number, Pe_{AB} is less than 10,000. This Peclet number is equal to the product of Reynolds and Schmidt numbers (i.e.)

$$Pe_{AB} = Re Sc \text{ ----- (4.46)}$$

For Peclet numbers greater than 10,000, the relation given by Levich is useful

$$Sh = 1.01 Pe_{AB}^{1/3} \text{ ----- (4.47)}$$

The relation given by Froessling

$$Sh = 2 + 0.552 Re^{1/2} Sc^{1/3} \text{ ----- (4.48)}$$

correlates the data for mass transfer into gases for at Reynold's numbers ranging from 2 to 800 and Schmidt number ranging 0.6 to 2.7.

For natural convection mass transfer the relation given by Schutz

$$Sh = 2 + 0.59 (Gr_{AB} Sc)^{1/4} \text{ ----- (4.49)}$$

is useful over the range

$$2 * 10^8 < Gr_{AB} Sc < 1.5 * 10^{10}$$

6. The mass flux from a 5 cm diameter naphthalene ball placed in stagnant air at 40°C and atmospheric pressure, is $1.47 * 10^{-3} \text{ mol/m}^2 \cdot \text{sec}$. Assume the vapor pressure of naphthalene to be 0.15 atm at 40°C and negligible bulk concentration of naphthalene in air. If air starts blowing across the surface of naphthalene ball at 3 m/s by what factor will the mass transfer rate increase, all other conditions remaining the same?

For spheres :

$$Sh = 2.0 + 0.6 (Re)^{0.5} (Sc)^{0.33}$$

Where Sh is the Sherwood number and Sc is the Schmids number. The viscosity and density of air are $1.8 * 10^{-5} \text{ kg/m.s}$ and 1.123 kg/m^3 , respectively and the gas constant is $82.06 \text{ cm}^3 \cdot \text{atm/mol.K}$.

Calculations:

$$Sh = \frac{k_c L}{D_{AB}} \text{ where L is the characteristic dimension for sphere L =}$$

Diameter.

$$Sc = \frac{\mu}{\rho D_{AB}}$$

$$R_c = \frac{Dv\rho}{\mu}$$

$$\text{Mass flux, } N_A = K_c \Delta c \text{ -----(1)}$$

$$Sh = 2.0 + 0.6 (Re)^{0.5} (Sc)^{0.33}$$

$$\frac{k_c D}{D_{AB}} = 2.0 + 0.6 \left(\frac{Dv\rho}{\mu} \right)^{0.5} \left(\frac{\mu}{\rho D_{AB}} \right)^{0.33} \text{ -----}$$

(2)

$$\text{also } N = K_G \Delta \bar{p}_A$$

$$\text{Therefore } \frac{k_c}{RT} = K_G$$

Given:

$$N = 1.47 * 10^{-3} \frac{\text{mol}}{\text{m}^2 \cdot \text{sec}} = \frac{K_c}{RT} \Delta \bar{p}_A$$

$$\frac{k_c}{RT} \left(\frac{0.15}{1} - 0 \right) = 1.47 * 10^{-3} * 10^{-4} \frac{\text{mol}}{\text{cm}^2 \cdot \text{sec}}$$

$$k_c = \frac{1.47 * 10^{-7}}{0.15} * 82.06 * (273 + 40)$$

$$= 0.0252 \text{ cm/sec}$$

$$k_c = 2.517 * 10^{-4} \text{ m/sec -----(3)}$$

Estimation of D_{AB} :

From (2),

$$\frac{2.517 * 10^{-4} * 5 * 10^{-2}}{D_{AB}} = 2 \text{ (since } v = 0 \text{)}$$

$$\text{Therefore } D_{AB} = 6.2925 * 10^{-6} \text{ m}^2/\text{sec}.$$

And

$$\frac{k_c * 5 * 10^{-2}}{6.2925 * 10^{-6}} = 2 + 0.6 \left(\frac{5 * 10^{-2} * 3 * 1.123}{1.8 * 10^{-5}} \right)^{0.5} \left(\frac{1.8 * 10^{-5}}{1.123 * 6.2925 * 10^{-6}} \right)^{0.33}$$

$$7946 k_c = 2 + 0.6 * (96.74) * (1.361)$$

$$k_c = 0.0102 \text{ m/sec.} \text{ ----- (4)}$$

$$\frac{(4)}{(3)} \Rightarrow \frac{N_{A2}}{N_{A1}} = \frac{0.0102}{2.517 * 10^{-4}} = 40.5$$

Therefore, rate of mass transfer increases by 40.5 times the initial conditions.

2.6.3 Single Cylinder

Several investigators have studied the rate of sublimation from a solid cylinder into air flowing normal to its axis. Bedingfield and Drew correlated the available data in the form

$$\frac{k_G P Sc^{0.56}}{G_m} = 0.281 (Re')^{-0.4} \text{ ----- (4.50)}$$

which is valid for $400 < Re' < 25000$

and $0.6 < Sc < 2.6$

Where Re' is the Reynold's number in terms of the diameter of the cylinder, G_m is the molar mass velocity of gas and P is the pressure.

2.6.4 Flow Through Pipes

Mass transfer from the inner wall of a tube to a moving fluid has been studied extensively. Gilliland and Sherwood, based on the study of rate of vapourization of nine different liquids into air given the correlation

$$Sh \frac{p_{B,lm}}{P} = 0.023 Re^{0.83} Sc^{0.44} \text{ ----- (4.51)}$$

Where $p_{B,lm}$ is the log mean composition of the carrier gas, evaluated between the surface and bulk stream composition. P is the total pressure. This expression has been found to be valid over the range

$$\begin{aligned} 2000 < Re < 35000 \\ 0.6 < Sc < 2.5 \end{aligned}$$

Linton and Sherwood modified the above relation making it suitable for large ranges of Schmidt number. Their relation is given as

$$Sh = 0.023 Re^{0.83} Sc^{1/3} \text{ ----- (4.52)}$$

and found to be valid for

$$\begin{aligned} & 2000 < Re < 70000 \\ \text{and } & 1000 < Sc < 2260 \end{aligned}$$

8. A solid disc of benzoic acid 3 cm in diameter is spin at 20 rpm and 25°C. Calculate the rate of dissolution in a large volume of water. Diffusivity of benzoic acid in water is $1.0 \times 10^{-5} \text{ cm}^2/\text{sec}$, and solubility is 0.003 g/cc. The following mass transfer correlation is applicable:

$$Sh = 0.62 Re^{1/2} Sc^{1/3}$$

Where $Re = \frac{D^2 \omega \rho}{\mu}$ and ω is the angular speed in radians/time.

Calculations:

$$\text{Dissolution rate} = N_A S \text{ -----(1)}$$

Where N_A = mass flux, and
 S = surface area for mass transfer

$$N_A = k_c (C_{As} - C_{A\infty}) \text{ ----- (2)}$$

Where C_{As} is the concentration of benzoic acid at the surface of the disc.

$C_{A\infty}$ is the concentration benzoic acid in water far from the surface of the disc.

Given:

$$Sh = 0.62 Re^{1/2} Sc^{1/3}$$

$$\text{(i.e.) } \frac{k_c D}{D_{AB}} = 0.62 \left(\frac{D^2 \omega \rho}{\mu} \right)^{1/2} \left(\frac{\mu}{\rho D_{AB}} \right)^{1/3} \text{ ----- (3)}$$

$$1 \text{ rotation} = 2 \pi \text{ radian}$$

$$\text{Therefore } 20 \text{ rotation per minute} = 20 \times 2 \pi \text{ radian/min}$$

$$= \frac{20}{60} \times 2 \pi \text{ radian/sec}$$

$$\text{For water } \rho = 1 \text{ g/cm}^3 \quad \mu = 1 \text{ centipoise} = 0.01 \text{ g/cm.sec.}$$

From (3),

$$\begin{aligned}
 k_c &= 0.62 D_{AB} \left(\frac{\omega \rho}{\mu} \right)^{\frac{1}{2}} \left(\frac{\mu}{\rho D_{AB}} \right)^{\frac{1}{3}} \\
 &= 0.62 * 1.0 * 10^{-5} * \left(\frac{(40 \pi / 60) * 1}{0.01} \right)^{\frac{1}{2}} \left(\frac{0.01}{1 * 1.0 * 10^{-5}} \right)^{\frac{1}{3}} \\
 &= 8.973 * 10^{-4} \text{ cm/sec.}
 \end{aligned}$$

From (2),

$$\begin{aligned}
 N_A &= 8.973 * 10^{-4} (0.003 - 0) \\
 &= 2.692 * 10^{-6} \text{ g/cm}^2 \cdot \text{sec}
 \end{aligned}$$

From (1),

$$\begin{aligned}
 N_A S &= N_A * (2\pi r^2) \\
 &= 2.692 * 10^{-6} * (2\pi * 1.5^2) \\
 &= 3.805 * 10^{-5} \text{ g/sec} \\
 &= 0.137 \text{ g/hr.}
 \end{aligned}$$

2.7 Mass transfer between phases:

Instead of a fluid in contact with a solid, suppose we now consider two immiscible fluids, designated 1 and 2, in contact with each other. If fluid 1 has dissolved in it a substance A that is also soluble in fluid 2, then as soon as the two fluids are brought together, substance A will begin to diffuse into fluid 2. As long as the two phases remain in contact, the transport of A will continue until a condition of equilibrium is reached.

The situation discussed here occurs in a variety of engineering processes such as gas absorption, stripping, and in liquid – liquid extraction. In all these separation processes, two immiscible fluids are brought into contact and one or more components are transferred from one fluid phase to the other. Concentration C_{A1} and C_{A2} are the bulk phase concentrations. C_{Ai} is the concentration of A at the interface, and N_A is the molar flux of A. For steady state conditions, we can define the flux of A as

$$N_A = k_{c1} (C_{A1} - C_{Ai}) = k_{c2} (C_{Ai} - C_{A2}) = K_c (C_{A1} - C_{A2}) \quad (1)$$

where k_c = individual mass transfer coefficient defined in terms of the concentration difference in a single phase.

K_c = overall mass transfer coefficient defined in terms of the overall difference in composition.

Equation (1) is analogous to that in heat transfer, where the individual coefficients h are related to the overall coefficient U .

From equation (1),

$$\frac{1}{k_{c1}} + \frac{1}{k_{c2}} = \frac{1}{K_c} \quad \text{_____} \quad (2)$$

In equation (1), the potential for mass transfer is exposed in terms of composition. However, this is not always the most convenient way to express it. For example, if fluid 1 is a gas and fluid 2 a liquid, as in gas absorption, the potential in gas phase is often expressed in terms of partial pressures, while that in the liquid phase may be expressed in terms of concentrations. The expression for the molar flux is then written for the individual phases as:

$$N_A = K_p (P_{AG} - P_{Ai}) = K_c (C_{Ai} - C_{AL}) \quad \text{_____}$$

(3)

where

k_p = individual mass transfer coefficient for the gas phase with the potential defined in terms of partial pressures.

P_{AG} , C_{AL} = partial pressure and concentration of A in the bulk gas and liquid phases, respectively.

P_{Ai} , C_{Ai} = partial pressure and concentration of A, respectively, at the interface.

At the interface, it is usually assumed the two phases are in equilibrium. This means that P_{Ai} and C_{Ai} are related by an equilibrium relationship such as 'Henry's law':

$$P_{Ai} = H C_{Ai} \quad \text{_____} \quad (4)$$

where H is Henry's law constant.

The flux N_A can also be expressed in terms of overall mass transfer coefficients as,

$$N_A = K_p (P_{AG} - P_{AE}) = K_c (C_{AE} - C_{AL}) \quad \text{_____} \quad (5)$$

where

K_p = overall mass transfer coefficient with the overall potential defined in terms of partial pressures.

K_c = overall mass transfer coefficient with the overall potential defined in terms of concentrations.

P_{AE} , C_{AE} = equilibrium composition.

P_{AE} is related to the bulk liquid composition C_{AL} AS

$$P_{AE} = H C_{AL} \quad \text{_____} \quad (6)$$

$$\text{similarly, } C_{AE} = \frac{P_{AG}}{H} \quad \text{_____} \quad (7)$$

The relationship between the individual and overall coefficients is readily obtained through the use of equations (3) to (7) as

$$\frac{1}{K_p} = \frac{1}{k_p} + \frac{H}{k_c} = \frac{H}{K_c} \quad \text{_____} \quad (8)$$

In many system, mass transfer resistance is mainly in one phase. For example, gases such as nitrogen and oxygen do not dissolve much in liquids. Their Henry's law constant H is very large, thus $K_c \approx k_c$ is a good approximation. In this case, the liquid phase controls the mass transfer process since mass transfer is slowest there.

2.8 Simultaneous Heat and Mass Transfer

Diffusional mass transfer is generally accompanied by the transport of energy, even with in an isothermal system. Since each diffusing constituent carries its own individual enthalpy, the heat flux at a given plane is expressed as

$$q = \sum_i N_i \bar{H}_i \quad \text{_____} \quad (1)$$

where q is the heat flux due to diffusion of mass past the given plane, and $\bar{H}_i$ is the partial molar enthalpy of constituent i in the mixture.

When there is a temperature difference, energy transfer also occurs by one of the three heat transfer mechanisms (conduction, convection, radiation) ; for example, the equation for energy transport by convection and molecular diffusion becomes

$$q = h \Delta T + \sum_i N_i \bar{H}_i \quad \text{_____} \quad (2)$$

If the heat transfer is by conduction, the first term on the right hand side of equation (2) becomes $k \Delta T / L$ where L is the thickness of the phase through which conduction takes place.

The most common examples of processes involving heat and mass transfer are condensation of mist on a cold surface and in wet bulb thermometer. There are a number of such processes involving simultaneous heat and mass transfer such as in formation of fog, and in cooling towers.

2.8.1 Condensation of vapor on cold surface:

A process important in many engineering processes as well as in day – to – day events involve the condensation of a vapor upon a cold surface. Examples of this process include “sweating” on cold water pipes and the condensation of moist vapor on a cold surface.

The process which involves a film of condensed liquid following down a cold surface and a film of gas through which the condensate is transferred by molecular diffusion. This process involves the simultaneous transfer of mass and energy.

The heat flux passing through the liquid film is given by

$$q = h_l (T_2 - T_3) \quad \text{_____} \quad (1)$$

This flux is also equal to the total energy transported by convection and molecular diffusion in the gas film.

$$\text{(i.e.,)} \quad q = h_c (T_1 - T_2) + N_A M_A (H_1 - H_2) \quad \text{_____} \quad (2)$$

where M_A is the molecular weight of the diffusing constituent A. H_1 and H_2 are enthalpies of the vapor at plane 1 and liquid at plane 2.

From equation (1) and (2)

$$q = h_l (T_2 - T_3) = h_c (T_1 - T_2) + N_A M_A (H_1 - H_2) \quad (3)$$

The molar flux N_A is calculated by diffusion through stagnant gas model as

$$N_A = \frac{-C D_{AB}}{1 - y_A} \frac{dy_A}{dz}$$

substituting the appropriate limits, the integral form of equation is

$$N_A = \frac{(C D_{AB})_{avg} (y_{A1} - y_{A2})}{(Z_2 - Z_1) y_{B,lm}} \quad (4)$$

2.8.2 The Wet – bulb Thermometer

The another example of simultaneous heat and mass transfer is that taking place in wet-bulb thermometer. This convenient device for measuring relative humidity of air consists of two conventional thermometer, one of which is clad in a cloth nick wet with water. The unclad dry-bulb thermometer measures the air's temperature. The clad wet-bulb thermometer measures the colder temperature caused by evaporation of the water.

We want to use this measured temperature difference to calculate the relative humidity in air. This relative humidity is defined as the amount of water actually in the air divided by the amount at saturation at the dry-bulb temperature. To find this humidity, we can write equation for the mass and energy fluxes as:

$$N_A = k_c (C_{Ai} - C_A) = k_y (y_{Ai} - y_A) \quad (1)$$

and $q = h (T_i - T) \quad (2)$

where C_{Ai} and C_A are the concentrations of water vapor at the wet bulb's surface and in the bulk of air, y_{Ai} and y_A are the corresponding mole fractions; T_i is the wet-bulb temperature, and T is the dry-bulb temperature. It can be noted that y_{Ai} is the value at saturation at T_i .

In the air-film surroundings the wet-bulb, the mass and energy fluxes are coupled as

$$N_A \lambda = -q \quad (3)$$

where λ is the latent heat of vaporization of water.

Thus,

$$k_y (y_{Ai} - y_A) \lambda = h (T - T_i)$$

Rearranging,

$$T_i = T - \frac{\lambda k_y}{h} (y_{Ai} - y_A) \quad (4)$$

From Chilton – colburn analogy,

$$\text{or } j_H = j_D$$

$$\frac{h}{\rho v^2 C_p} (\text{Pr})^{2/3} = \frac{k_c}{v_\alpha} (\text{Sc})^{2/3} \quad \text{----- (5)}$$

For gas $\text{Pr} \approx 1$ and $\text{Sc} \approx 1$.

Therefore equation (5) becomes,

$$\frac{k_c}{h} = \frac{1}{C_p} \quad (\text{as } k_y C \approx k_y \rho = k_c)$$

Therefore equation (4) becomes

$$T_i = T - \frac{\lambda}{C_p} (y_{Ai} - y_A)$$

where C_p is the humid heat of air. By similar method, the other industrial processes of importance involving simultaneous heat and mass transfer such as humidification and drying can be analysed.

14. Air at 1 atm is blown past the bulb of a mercury thermometer. The bulb is covered with a wick. The wick is immersed in an organic liquid (molecular weight = 58). The reading of the thermometer is 7.6 °C. At this temperature, the vapor pressure of the liquid is 5 kPa. Find the air temperature, given that the ratio of heat transfer coefficient to the mass transfer coefficient (psychrometric ratio) is 2 kJ/kg. Assume that the air, which is blown, is free from the organic vapor.

Solution:

For simultaneous mass and heat transfer, heat flux q and mass flux N_A are related as

$$q = N_A \lambda \text{ ----- (1)}$$

where λ is the latent heat of vaporization. Mass flux is given by

$$N_A = k_Y (Y'_\omega - Y') \text{ ----- (2)}$$

where

k_Y = mass transfer coefficient

Y'_ω = mass ratio of vapor in surrounding air at saturation; and

Y' = mass ratio of vapor in surrounding air.

Convective heat flux is given by

$$q = h (T - T_\omega) \text{ ----- (3)}$$

where

h = heat transfer coefficient;

T_w = wet bulb temperature of air; and

T = dry bulb temperature of air.

Substituting for N_A and q from equation (2) and equation (3) in equation (1),

$$h(T - T_w) = k_y (T_w - Y') \lambda$$
$$T - T_w = \frac{\lambda (Y_w' - Y')}{h/k_y} \text{----- (4)}$$

Given: $Y' = 0$; $\lambda = 360$ kJ/kg; $h/k_y = 2$ kJ/kg.K; and $T_w = 7.6^\circ\text{C}$

$$Y_w' = \frac{\text{kg organic vapor at saturation}}{\text{kg dry air}}$$
$$= \frac{5}{101.3 - 5} \frac{58}{29} = 0.1038$$

Substituting these in equation (4)

$$T - 7.6 = \frac{(360)(0.1038 - 0)}{2} = 18.69$$

$$T = 18.69 + 7.6 = 26.29^\circ\text{C}$$

Temperature of air = 26.29°C .